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Electron-ion collisional effect on Weibel instability in a Kappa distributed unmagnetized plasma

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Weibel instability has been investigated in the presence of electron-ion collisions by using standard Vlasov-Maxwell equations. The presence of suprathermal electrons has been included here by using Kappa distribution for the particles. The growth rate $\gamma$ of Weibel instability has been calculated for different values of spectral index $\kappa$, collision frequency $\nu_{ei}$, and temperature anisotropy parameter $\beta$. A comparative study between plasma obeying Kappa distribution and that obeying Maxwellian distribution shows that the growth of instability is higher for the Maxwellian particles. However, in the presence of collisions, the suprathermal particles result in lower damping of Weibel mode. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4870083]

I. INTRODUCTION

When intense ultrashort laser pulse interacts with a solid target, it is found that large magnetic field of the order of $10^8$ G is generated.\textsuperscript{1} It is a challenging task to understand the origin and evolution of large scale magnetic field in astrophysical environment and fast ignition scenarios. Weibel\textsuperscript{2} in his classic work (1959) showed that velocity anisotropy of electron may lead to spontaneous excitation of transverse electromagnetic wave. Achterberg\textsuperscript{3} suggested that filamentation and temperature anisotropy driven Weibel instability are responsible for generation of magnetic field in the early stage of universe, ultra-relativistic shocks associated with Gamma Ray Bursts, etc. In recent years, Weibel instability has been widely studied in laser-produced plasmas. Sandhu et al.\textsuperscript{4} have reported about their observation of generation of ultra-short (6ps) multi-Mega gauss (27 MG) magnetic pulse during interaction of intense laser pulse $(106 \text{ W cm}^{-2}, 100 \text{fs})$ with a solid target. In Fast Ignition (FI) scheme, the electron beam may have significant temperature anisotropy, which leads to the Weibel instability. When fast electrons move into a target, a relatively colder return current having much higher density is generated, and this return current may be a cause of Weibel instability. The magnetic field generated due to Weibel instability may play an important role in transport of fast electrons in the coronal plasma of FI targets.\textsuperscript{5} Okada et al.\textsuperscript{6} have demonstrated by using PIC simulation that Weibel instability leads to filamentation of fast electron current in FI scheme. Theoretical analysis and PIC simulation of Weibel instability in laser plasma have been reported in literature since last few years. Califano et al.\textsuperscript{7} have studied the nonlinear saturation of Weibel instability in an unmagnetized plasma. They have estimated the fraction of the kinetic energy of the counter-streaming electrons that is transformed to magnetic energy by Weibel instability. Role of collisions on Weibel instability has been investigated by many researchers. It is well known that collisions may play important role in FI and laser-solid interactions. Electron-ion binary collision is important in relativistic beam transportation process of FI scheme of Inertial Confinement Fusion (ICF).\textsuperscript{8} Karmakar et al.\textsuperscript{9} have developed a simplified model to study collisional Weibel instability. They have showed that the temperature of beam does not destroy Weibel instability even in the presence of collisions in the beam plasma system.

Non-Maxwellian plasma is widely found in space with suprathermal tails in electron or ion distribution. Energetic electrons may be generated in laser-plasma during beam-target interaction. Such energetic electrons are well fitted by the Kappa distribution. Kappa distribution has been used to explain different waves and instabilities, and the results are found in good quantitative agreement with observations, which indicate that Kappa distribution may be a more appropriate substitute of Maxwellian distribution in some circumstances. Summers and Thorne\textsuperscript{10} have introduced the modified plasma dispersion function, i.e., $Z_{\beta}(\zeta)$, which is analogous to the standard plasma dispersion function $Z(\zeta)$ based on the Maxwellian distribution. Gloeckler et al.\textsuperscript{11} have found that the Kappa distributions with $2 < \kappa < 6$ fit with the observations and satellite data of the solar wind. Maksimovic\textsuperscript{12} has worked on a kinetic model of the solar wind with Kappa distribution functions in the corona. Zaheer\textsuperscript{13} has worked on Weibel instability with non-Maxwellian distribution functions and has presented a comparative study between the growth rates for particles with Maxwellian, Kappa, and the generalized $(r,q)$ distribution function. Recently, Mahdavi\textsuperscript{14} has investigated the effect of Coulomb collision of electron-ion on the Weibel instability with bi-Maxwellian distribution function.

A literature review has revealed that Weibel instability has been widely discussed, both in laser-plasma and in astrophysical situations. However, to our knowledge, the effect of collisions on Weibel instability using suprathermal electrons has not been studied so far. Suprathermal electrons are widely present in astrophysical situations as well as in some laser-plasma experiments. It is therefore very much relevant and important to investigate the generation of magnetic field
via Weibel instability due to the suprathermal electrons. In this paper, we present an analytic theory of Weibel instability using non-relativistic Kappa distribution taking into account the effect of electron-ion collision.

II. MATHEMATICAL MODEL

We apply the standard Vlasov-Maxwell formalism of kinetic theory to combine the non-relativistic Vlasov equation having a collision term with the Maxwell equations

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + q \left( \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \frac{\partial f}{\partial \mathbf{v}} = -\nu_{ei} (f - f_0),$$

(1)

$$\nabla \times \mathbf{E} = \frac{-1}{c} \frac{\partial \mathbf{B}}{\partial t},$$

(2)

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{j}.$$  

(3)

In the above equations, $f$ is the electron distribution function at position $\mathbf{r}$ and momentum $\mathbf{p}$ at time $t$, $f_0$ is the total equilibrium distribution function, $\mathbf{J}$, $c$, $\mathbf{E}$, and $\mathbf{B}$ have their usual meanings. The momentum $\mathbf{p}$ is related to the rest mass $m$ and velocity $\mathbf{v}$ of the electron by $\mathbf{p} = m \mathbf{v}$, and $\nu_{ei}$ is the electron-ion collision frequency.

Here, we consider a model where the electromagnetic wave propagates in the direction $\hat{k} = k\hat{e}_k$ and the total equilibrium distribution function $f_k$ is described by the three-dimensional kappa distribution function

$$f_k = \frac{1}{\pi^{3/2} \theta_k^{3/2} } \frac{\Gamma(k+1)}{ \kappa^{3/2} \Gamma(k-1/2) } \left[ 1 + \frac{v_{k+}^2}{\kappa \theta_k^2} + \frac{v_{k-}^2}{\kappa \theta_k^2} \right]^{-(k-1)},$$

(4)

where $k$ is the spectral index; $\theta$ is the thermal speed and is related to the particle temperature $T$ by

$$\theta_k = \left( \frac{2k - 3}{k} \right)^{1/2} (v_{k+}^2)^{1/2}; \quad \theta_k = \left( \frac{2k - 3}{k} \right)^{1/2} (v_{k-}^2)^{1/2},$$

(5)

where

$$\frac{v_{k+}^2}{m} = \frac{T_{k+}}{m}.$$  

(6)

The spectral index $k$ is constrained to $k > \frac{3}{2}$ due to normalization and the definition of the temperature. $\Gamma$ is the gamma function and $f_k$ has been normalized so that $\int f_k d^3 v = 1$.

Using Eqs. (1)–(3), the dispersion relation for an unmagnetized plasma in cylindrical coordinates is given as

$$\omega^2 - c^2 k^2 - \omega_{pe}^2 + \pi \omega_{pe}^2 \left( \frac{k}{m} \right) \int_{-\infty}^{+\infty} \frac{dp_{\perp}}{\omega' - k v_{\perp}}$$

$$\times \int_0^\infty p_{\perp}^2 \left( \frac{\partial f_0}{\partial p_{\perp}} \right) dp_{\perp} = 0,$$

(7)

where $\omega$ and $k$ are the frequency and wave number of wave instability, respectively, $\omega_{pe}$ is the electron plasma frequency, and $\omega' = \omega + \nu_{ei}$. The derivative of Eq. (4) with respect to the parallel momentum can be obtained as follows:

$$\frac{\partial f_0}{\partial p_{\parallel}} = \frac{2e_{\parallel}}{m \pi^{3/2} \theta_k^2 \theta_k^{3/2} \kappa^{3/2} \Gamma(k-1/2)} \left[ 1 + \frac{v_{k+}^2}{\kappa \theta_k^2} + \frac{v_{k-}^2}{\kappa \theta_k^2} \right]^{-(k-2)}.$$  

(8)

Substituting above relation in Eq. (7), we obtain

$$\omega^2 - c^2 k^2 - \omega_{pe}^2 \left( 1 - \frac{T_{\perp}}{T_{\parallel}} \right) + \frac{\pi \omega_{pe}^2}{\sqrt{\pi}} \left( T_{\perp} / T_{\parallel} \right) \left( \frac{\Gamma(k)}{k^{1/2} \Gamma(k-1/2)} \right)$$

$$\times (\xi) \int_{-\infty}^{+\infty} \frac{(1 + x^2/\kappa)^{-k}}{(x - \xi)} dx = 0.$$  

(9)

where $x = \frac{p_{\perp}}{m \theta_k}$ and $\xi = \frac{\omega'}{k v_{\perp}}$.

Here, we use the Plemelj formula

$$\int_{-\infty}^{+\infty} \frac{(1 + x^2/\kappa)^{-k}}{(x - \xi)} dx = P \int_{-\infty}^{+\infty} \frac{(1 + x^2/\kappa)^{-k}}{(x - \xi)} dx$$

$$+ i \pi \left( 1 + \frac{\xi^2}{\kappa} \right)^{-k}.$$  

(10)

Integrating the principal part, we obtain

$$P \int_{-\infty}^{+\infty} \frac{(1 + x^2/\kappa)^{-k}}{(x - \xi)} dx \sqrt{\pi k^{1/2} \Gamma(k-1/2)}$$

$$\times \left( 1 + \frac{k \Gamma(k-3/2)}{\Gamma(k-1/2)} \right).$$  

(13)

Considering the dispersion relation in Eq. (11), we define a new plasma dispersion function

$$Z_k(\xi) = \frac{1}{\sqrt{\pi} k^{1/2} \Gamma(k-1/2)} \int_{-\infty}^{+\infty} \frac{(1 + x^2/\kappa)^{-k}}{(x - \xi)} dx.$$  

(14)

Thus, Eq. (11) can be rewritten as
\[
\omega^2 - c^2 k^2 - \omega_{p}^2 \left(1 - \frac{T_i}{T_i} \right) + \omega_{pe}^2 \left( \frac{T_i}{T_i} \right) (\xi) Z_1^\alpha(\xi) = 0. \tag{15}
\]

Assuming \(\xi < 1\), the solution for \(Z_1^\alpha(\xi)\) can be obtained for integral values of \(\alpha\). Thus, for \(\alpha = 3, 4, 5\), the following three \(Z\)-functions can be obtained, respectively:

\[
Z_1^\alpha(\xi) = \xi (-1.66 - 0.370 \xi^2 - \cdots) + i (1.539 - 1.539 \xi^2 - \cdots), \tag{16}
\]

\[
Z_1^\alpha(\xi) = \xi (-1.75 - 0.437 \xi^2 - \cdots) + i (1.6 - 1.6 \xi^2 + \cdots), \tag{17}
\]

\[
Z_1^\alpha(\xi) = \xi (-1.8 - 0.48 \xi^2 - \cdots) + i (1.635 - 1.635 \xi^2 + \cdots). \tag{18}
\]

Thus, using the approximation \(\omega^2 \ll k^2 c^2\), we obtain the following dispersion relations corresponding to \(\alpha = 3, 4, 5\):

For \(\alpha = 3\), we get

\[
c^2 k^2 + \omega_{p}^2 \left(1 - \frac{T_i}{T_i} \right) \left(1 - 1.66 \xi^2 \right) - i \omega_{pe}^2 \left( \frac{T_i}{T_i} \right) 1.539 \xi = 0. \tag{19}
\]

Neglecting the term \(1.66 \xi^2\), we obtain

\[
Im \omega' = -(0.649) \left( \frac{k \theta_i}{\omega_{pe}} \right) \left( \frac{T_i}{T_i} \right) \left\{ c^2 k^2 + \omega_{p}^2 \left(1 - \frac{T_i}{T_i} \right) \right\} \tag{20}
\]

Similarly for \(\alpha = 4, 5\), we get

\[
Im \omega' = -(0.625) \left( \frac{k \theta_i}{\omega_{pe}} \right) \left( \frac{T_i}{T_i} \right) \left\{ c^2 k^2 + \omega_{p}^2 \left(1 - \frac{T_i}{T_i} \right) \right\} \tag{21}
\]

and

\[
Im \omega' = -(0.611) \left( \frac{k \theta_i}{\omega_{pe}} \right) \left( \frac{T_i}{T_i} \right) \left\{ c^2 k^2 + \omega_{p}^2 \left(1 - \frac{T_i}{T_i} \right) \right\}. \tag{22}
\]

Considering \(\omega = \omega_r + i \omega_i\), we can have \(\omega' = \omega_r + i (\omega_i + \nu_{ei})\). Thus, from Eq. (20) we obtain

\[
\gamma = (0.649) (k \theta_i) \left\{ 1 - \frac{1 + c^2 k^2 / \omega_{p}^2}{(T_i/T_i)} \right\} - \nu_{ei}. \tag{23}
\]

Similarly, from Eqs. (21) and (22), we get

\[
\gamma = (0.625) (k \theta_i) \left\{ 1 - \frac{1 + c^2 k^2 / \omega_{p}^2}{(T_i/T_i)} \right\} - \nu_{ei}, \tag{24}
\]

\[
\gamma = (0.611) (k \theta_i) \left\{ 1 - \frac{1 + c^2 k^2 / \omega_{p}^2}{(T_i/T_i)} \right\} - \nu_{ei}. \tag{25}
\]

In Eqs. (23)–(25), the first part refers to the growth rate for collisionless plasma and the second part refers to the collision frequency. With the increase of the collision frequency \(\nu_{ei}\), the growth rate \(\gamma\) decreases and thus leading towards the damping of the wave instability.

The Coulomb electron-ion collisional frequency for an anisotropic Maxwellian plasma distribution \((T_i \neq T_i)\) can be obtained as

\[
\nu_{ei}(T_{\perp}, T_i) = \frac{e^2 \epsilon_0}{12 \sqrt{3 \pi} \epsilon_0 m_i T_i^{1/2}} \ln \Lambda. \tag{26}
\]

The collisional frequency is modified for the particles obeying Kappa distribution and is dependent on the value of \(\kappa\). Each electron in the distribution is losing momentum to the ions at a rate given by the collision frequency

\[
\nu_{p}(v_{\perp}, v_{||}) = n_e \frac{v_e}{2} 4 \pi (m_e + m_i) \ln \Lambda. \tag{27}
\]

Thus, the frequency of momentum loss averaged over Kappa distributed particles is

\[
\bar{\nu}_{ei} = - \frac{1}{p} \frac{dp}{dt} = - \frac{1}{p} \int_0^\infty f_e(v_{\perp}, v_{||}) \nu_{p}(v_{\perp}, v_{||}) m_e v_{||} d^3 v_{\perp}, \tag{28}
\]

where \(p\) represents the momentum of the particles.

Carrying out the above integral, the collision frequency is obtained as

\[
\bar{\nu}_{ei} = \Gamma (\kappa + 1) \frac{\sqrt{\kappa} \epsilon_0}{\sqrt{\pi} \Gamma (\kappa - 1/2)} \frac{n_e}{8} \frac{v_e}{2} 4 \pi (m_e + m_i) \ln \Lambda. \tag{29}
\]

It is obvious from the above relation that collision frequency for particles obeying Kappa distribution differs from that of Maxwellian distribution and is dependent on the value of \(\kappa\). It is seen that collision frequency increases with \(\kappa\) and is less for Kappa distributed particles than that of the Maxwellian particles. It is therefore justified to use appropriate collision frequency for such Kappa distributed particles.

Here, we define a new parameter \(\beta\) such that \(\beta = T_i / T_i\). The quantity \(\beta\) gives a measure of the temperature anisotropy. Higher value of \(\beta\) indicates that the plasma temperature is highly anisotropic, while for a low value of \(\beta\) the temperature anisotropy is less. Thus, for \(\kappa = 3, 4, 5\), Eqs. (23)–(25) can be rewritten as

\[
\gamma = (0.649) (k \theta_i) \left\{ 1 - \frac{1 + c^2 k^2 / \omega_{p}^2}{(T_i/T_i)} \right\} - \nu_{ei}. \tag{30}
\]
\[
\gamma = (0.625)(k \theta_j) \left\{ 1 - \frac{(1 + c^2 k^2 / \omega_{pe}^2)}{\beta} \right\} - \nu_{ci},
\]
(31)

\[
\gamma = (0.611)(k \theta_j) \left\{ 1 - \frac{(1 + c^2 k^2 / \omega_{pe}^2)}{\beta} \right\} - \nu_{ci},
\]
(32)

Thus, from the above equations, it can be observed that the increase or decrease of \( \beta \) leads to the increase or decrease of growth rate of Weibel instability, respectively.

### III. RESULTS AND DISCUSSION

Using standard Vlasov-Maxwell equations, we have derived the dispersion relation for an unmagnetized plasma in the presence of electron-ion collisions. Here, we have analytically calculated the growth rate for Weibel instability by using the Kappa distribution and have done a comparative study between the Weibel growth rate for Kappa and Maxwellian distributions.\(^1\) We are interested to see the existence of Weibel instability in presence of collisions and the collisional effects on the growth rate for various values of spectral index \( \kappa \) and temperature anisotropy parameter \( \beta \). The normalized growth rate is plotted across normalized wave vector \( k \) for different collision frequencies \( \nu_{ci} \) in Fig. 1. The collision frequency can be varied by varying \( T_\perp \) and \( T_\parallel \) in equal proportions so that \( \beta \) remains unchanged. Growth rate decreases with an increase in \( \nu_{ci} \). Even in the presence of collisions, Weibel instability persists for large values of anisotropy in temperatures.

Fig. 2(a) displays the effect of suprathermal electrons on growth rate for different values of \( \kappa \). Since, collision frequency depends on the quantum of suprathermal electrons, it varies with the variation in the value of \( \kappa \). The collision frequency is high for large values of \( \kappa \). Fig. 2(a) reveals that in spite of having high collision frequency, growth rate increases with the increase in value of \( \kappa \) and is maximum for particles obeying Maxwellian distribution function. Suprathermal electrons contribute less to the generation of magnetic field via Weibel instability as compared to the Maxwellian ones.
Fig. 2(b) shows the variation of growth rate for different values of $\kappa$ at a reduced value of temperature anisotropy ($\beta = 20$). At a low value of $k$, the growth rate curves intersect with each other and the Weibel mode gets damped for higher values of $k$ beyond the point of intersection. The point of intersection shifts towards lower values of $k$ as the value of $\beta$ is reduced. Damping occurs at higher values of $k$ due to the reduction in the value of temperature anisotropy and due to the presence of collisions. Weibel instability is driven by the free energy caused by anisotropy in average kinetic energy, which is clearly revealed from the comparison of Figs. 2(a) and 2(b). Higher values of $\kappa$ lead to greater damping of Weibel instability, and it is highest for particles obeying Maxwellian distribution function. Thus, it can be concluded from this analysis that even in the presence of collisions, Weibel instability persists to a greater extent for suprathermal electrons since they suffer less damping as compared to the Maxwellian ones.

We further reduce the temperature anisotropy to $\beta = 2.5$ as shown in Fig. 2(c). Irrespective of the type of distribution function, Weibel instability shows damping for the entire range of $k$, which indicates that significant amount of temperature anisotropy is an essential factor for the growth of this instability in the presence of collisions.

**IV. CONCLUSION**

We have calculated the growth rate of Weibel instability with Kappa distribution in presence of electron-ion collisions in this manuscript. The presence of plasma particles, which deviate from Maxwellian distribution, results in lower growth rate than Maxwellian particles. The conclusions from the above discussions can be summarized as follows:

(i) Weibel mode exhibits growth for low values of collision frequency. Growth rate is higher for Maxwellian plasma particles.

(ii) The electron-ion collision frequency is affected by the spectral index $\kappa$. Suprathermal electrons experience less collision than the Maxwellian ones where the other parameters remain unchanged.

(iii) In presence of collisions, Weibel mode shows damping. The damping is high for a plasma with Maxwellian distribution function. For the same set of plasma parameters, Weibel instability persists to a greater extent for suprathermal electrons.

(iv) With decrease in temperature anisotropy, the point of intersection of the growth rate curves for different spectral indices shifts towards lower values of $k$.

(v) As the temperature anisotropy parameter $\beta$ increases, the normalized value of $k$ corresponding to maximum growth rate shifts towards higher values.