Sub-ion-scale magnetic spectra in kinetic Alfvén wave turbulence: phenomenology and FLR-Landau fluid simulations

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Outline

1. **KAW turbulence**: solar wind observations and existing numerical simulations
   Main questions concerning the **slope of magnetic spectra**

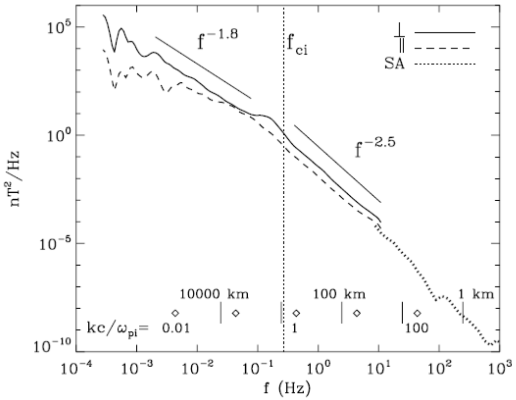
2. **Reduced models**: their properties and limitations

3. **FLR-Landau fluid model**:
   brief presentation
   numerical simulations

4. **Phenomenological model** for the spectrum of KAW turbulence

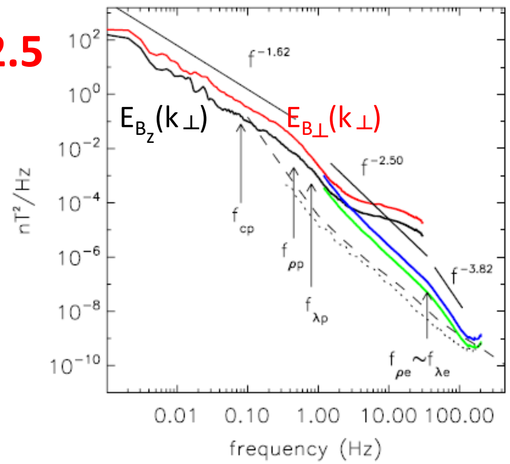
5. Conclusions and perspectives

Spectral exponent at sub-ion scales, excluding the transition range

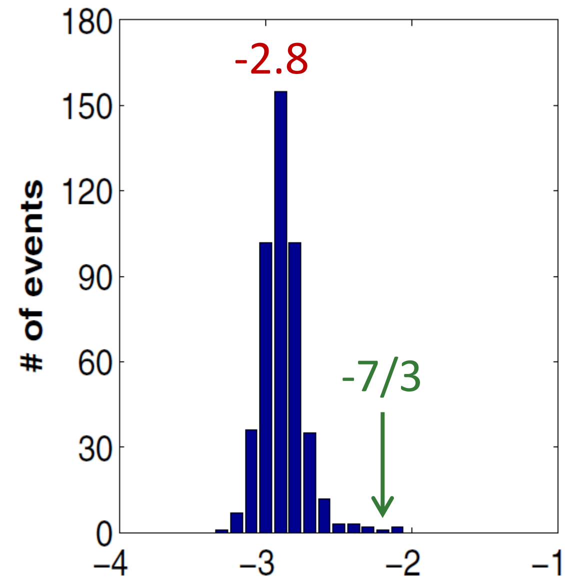


Alexandrova et al., Ann. Geophys. 26, 3585 (2008)

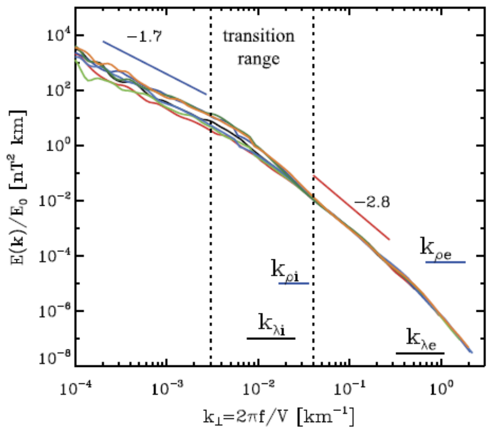
-2.5



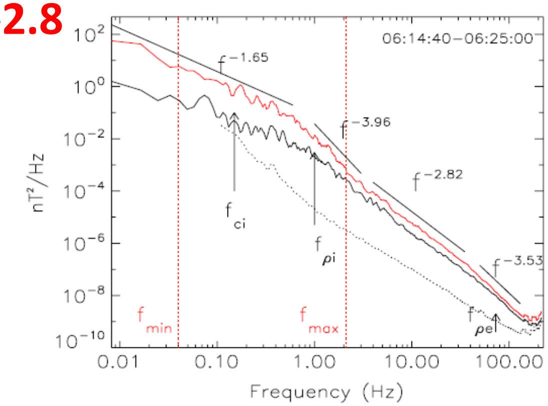
Sahraoui et al., PRL 102, 232202 (2009)



Sahraoui et al., ApJ **777**, 15, 2013

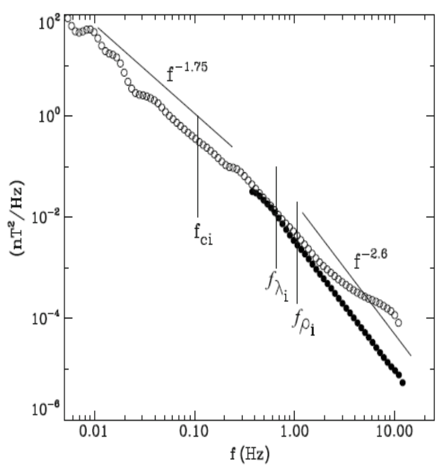


-2.8



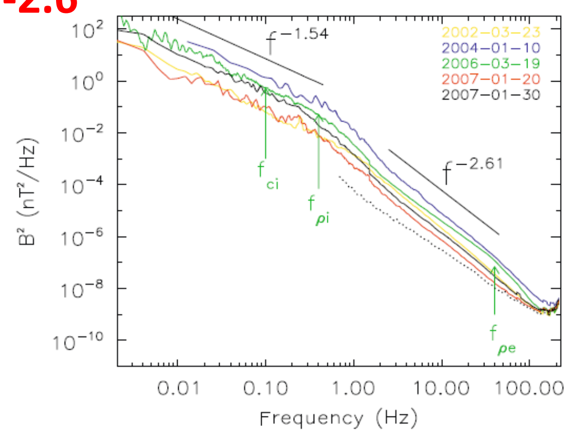
Sahraoui et al., PRL 105, 131101 (2010)

"the slopes of the spectra in the dispersive range (i.e., [$f_{\rho i}$, $f_{\rho e}$]) cover the domain $\sim [-2.5,-3.1]$ with a peak at ~ -2.8".



Alexandrova et al., Space Sci. Rev. 178, 101 (2013)

-2.6



Sahraoui et al., Planet. Space Sci. 59, 585 (2011)

While inertial range slopes: -1.63±0.14 (Smith at al. ApJ **645** L85 (2006) using ACE)

Magnetic spectrum in the solar wind
(Cluster observations)

-7/3: strong turbulence phenomenology (critical balance)

3D Electron-MHD in the presence of a strong magnetic field

*(Meyrand & Galtier, PRL **111**, 264501, 2013)*

Existence of a $k_\perp^{-8/3}$ spectral range

2D simulations in the plane perpendicular to the ambient field

Hybrid-PIC *(Franci et al., ApJL.. **804**, L39, 2015)*

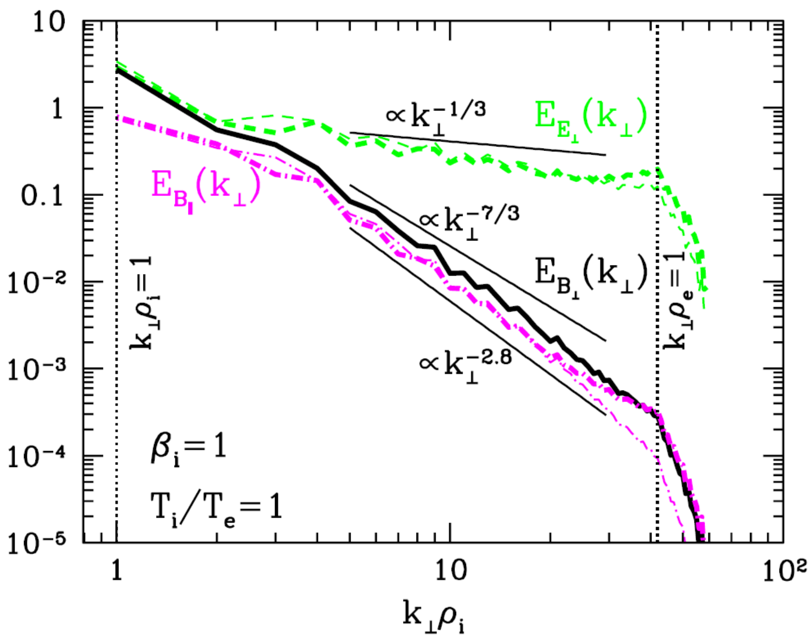
-5/3 spectrum at the MHD scales

-3 spectrum at the sub-ion scales

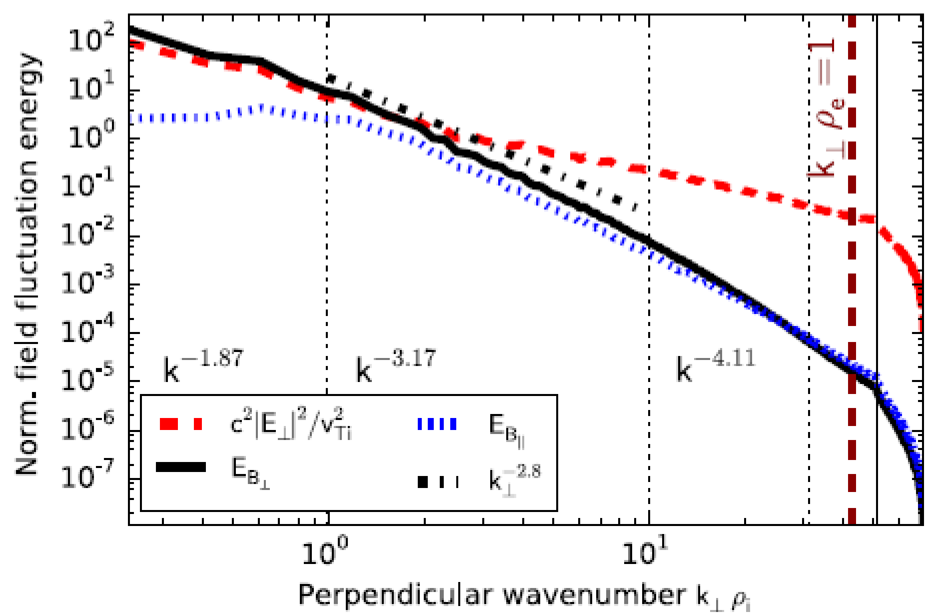
Hybrid-eulerian *(Cerri et al. ApJL 2016)*

B slope in sub-ion range between -8/3 and -3

Gyrokinetic simulations



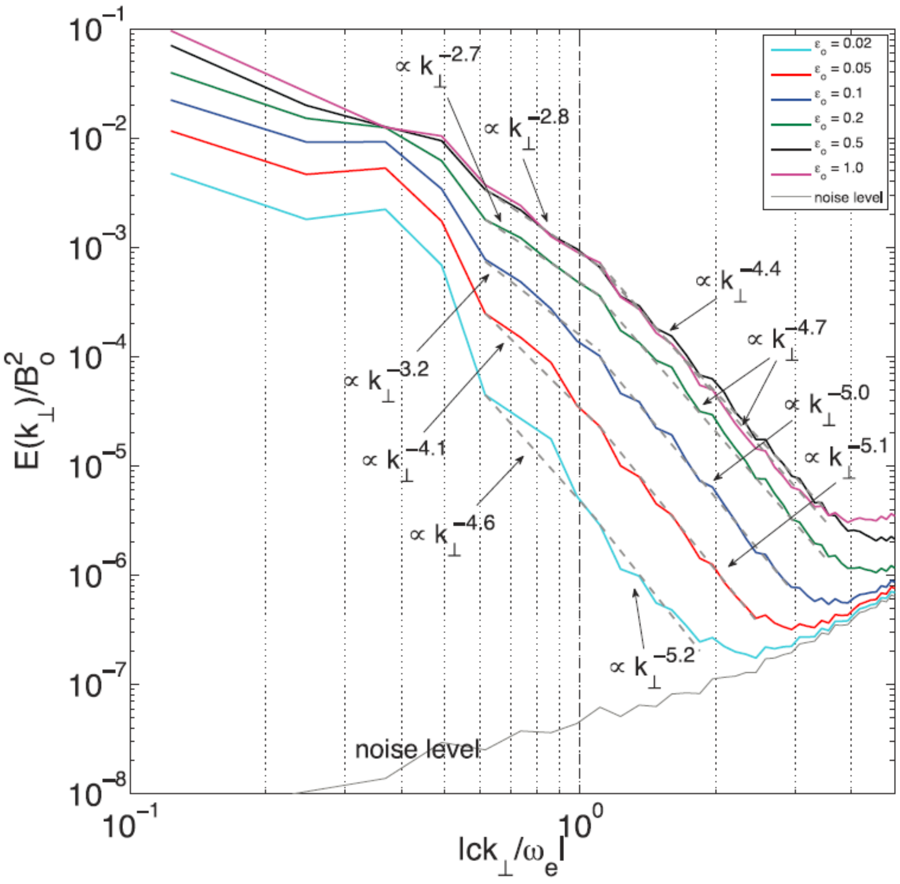
*(Howes et al., PRL **107**, 035004, 2011)*



*(Told et al., PRL **115**, 025003, 2015)*

3D full PIC whistler mode simulations

with various level of energy fluctuations (Gary et al., ApJ **755**, 142, 2012)



"**Increasing initial fluctuation amplitudes** over $0.02 < \varepsilon_0 < 0.50$ yields … a **consistent decrease in the slope of the spectrum** at $k_\perp c/\omega_e < 1$".

In apparent contrast to solar wind observations of Smith et al. (2006), Bruno et al. ApJL (2014).

(several parameters probably simultaneously changed and/or problem with definition of fluctuation amplitude)

Main points to understand, focusing on sub-ion spectral slopes

1. The observed spectra are **steeper than the -7/3 slope** predicted by most theories based on critical balance arguments.

2. Except in simpler models, the slopes display a rather **large scatter**.

Questions:

- What is the **correlation between the spectral slopes** and:

  - the **amplitude** of magnetic field fluctuations
  (to be defined properly)

  - the strength or **transfer rate of the turbulence**
  (as e.g. defined by extensions of Karman-Howarth equation as in Banerjee & Galtier PRE **87**, 013019 (2013))

  - the **beta** parameter

Reduced models

Need to perform large-scale simulations aiming at testing theories.

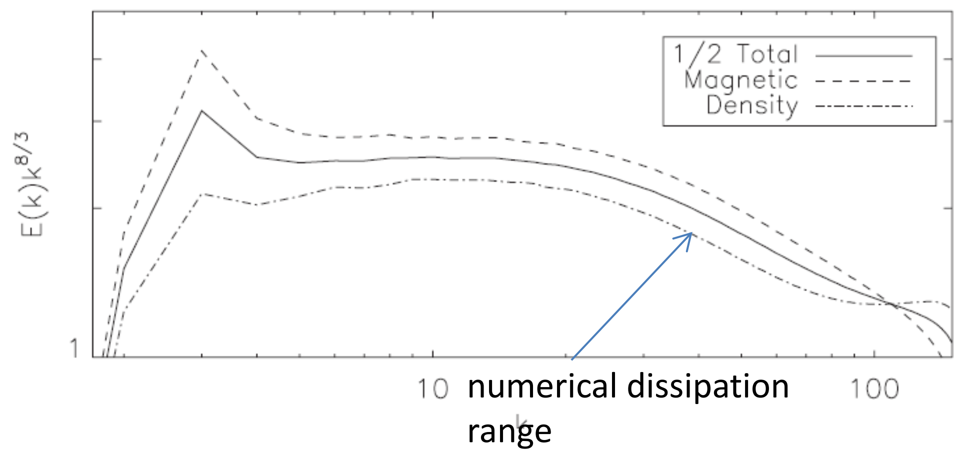
Such simulations have been done using a semi-phenomenological model assuming Boltzmanian ions and electrons:

*Boldyrev & Perez , ApJL, **758**, L44, 2012; see also Schekochihin et al. ApJ Supp. **182**, 310 (2009)*

$$\partial_t \psi + \nabla_\| n = 0, \qquad \nabla_\| = \nabla_z + \hat{z} \times \nabla\psi \cdot \nabla_\perp$$

$$\partial_t n - \nabla_\| \nabla_\perp^2 \psi = 0 \qquad \mathbf{b}_\perp = \hat{z} \times \nabla\psi$$

$$E = \int (|\nabla\psi|^2 + n^2) d^3x$$ is conserved.



Power counting gives exponent -7/3
but numerics suggests -8/3 ≈ 2.7
(viewed as intermittency corrections)

Spectrum independent of simulation parameters

More refined gyrofluid models

Rather rigorous fluid models can be derived from the gyrokinetic equation.

Few contain enough ingredients for this study (e.g. allow for $B_{\|}$ fluctuations)

One example is the one by Brizard : PoF 4, 1213 (1992).

Despite some shortcomings this model constitutes an interesting starting point to derive limiting equations valid for scales large compared with the electron Larmor radius and small compared with the ion Larmor radius (c.f. poster by E. Tassi et al.).

Interestingly such a model can also be derived from the FLR-Landau fluid model (see below).
Provides a generalization of Boldyrev's model with dynamical electron pressure equations.

BUT :

gyroaveraging (or cancellations of fluid quantities with FLR corrections in the FLR-LF model)

➔ Ion velocities and ion temperature fluctuations become subdominant at small scales

➔ Ignores ion Landau damping which turns out to be an important ingredient.

Influence of Landau damping

*(Howes et al. JGR 113, A05105, 2008; PoP **18**, 102305, 2011):*

Balance between energy transfer and Landau dissipation:

leads essentially to energy flux $\epsilon \sim \exp(-\lambda k_\perp)$ and $E(k_\perp) \sim k_\perp^{-7/3} \exp(-2\lambda k_\perp/3)$

For appropriate parameters gives the impression of a steeper power law.

Revised version *(Passot & Sulem Ap.J. Lett. **812**: L37, 2015)* predicts a **non-universal correction to the power-law exponent**.

Need to include both ion and electron Landau damping

➔**Turn to the FLR-Landau fluid model** to perform runs with varying parameters

FLR-Landau fluid

Fluid model retaining *Hall effect, Landau damping* and *ion finite Larmor radius (FLR) corrections in the sub-ion range*. Electron FLR corrections and electron inertia neglected.
(Extension of Landau fluid for MHD scales, *Snyder, Hammett & Dorland, Phys. Plasmas* ***4****, 3974, 1997).*

The fluid hierarchy for the gyrotropic moments is closed by evaluating the gyrotropic 4th rank cumulants and the non-gyrotropic contributions to all the retained moments, in a way **consistent with the linear kinetic theory, within a low-frequency asymptotics.**

In brief, consider the expressions of the various moments provided by the low-frequency linear kinetic theory, combine them to eliminate as much as possible the plasma dispersion fonctions $Z_r(\mathbf{k},\omega)$.
When not possible, use suitable Padé approximants.

The model reproduces dispersion and damping rate of low-frequency modes at the sub-ion scales.

Passot & Sulem, Phys. Plasmas ***14****, 082502, (2007); Passot, Sulem & Hunana, Phys. Plasmas* ***19****, 082113, (2012);*
Sulem & Passot, J. Plasma Phys. ***81*** *(1), 32810103 (2015)*

First 3D FLR-LF simulations of turbulence at ionic scales presented in
Passot, Henri, Laveder & Sulem, Eur. Phys. J. D. ***68****, 207, 2014.*
see also
Sulem, Passot, Laveder & Borgogno, ApJ ***816****:66 (2016).*

Alfvenic turbulence

The system is driven by a random forcing

$$F_i(t, \mathbf{x}) = \sum_{1<n<N} F^0_{i,n} \cos(\omega_{KAW}(\boldsymbol{k}_n)t - \boldsymbol{k}_n \cdot \boldsymbol{x} + \phi_{i,n})$$

KAW frequency of wavevector $\boldsymbol{k}_n$
Propagation angle : 80° - 86°

KAWs are generated by resonance

Driving is turned on (resp. off) when the sum of kinetic and magnetic energies is below (resp. above) a prescribed threshold: **prescribed amplitude of the turbulence fluctuations**.

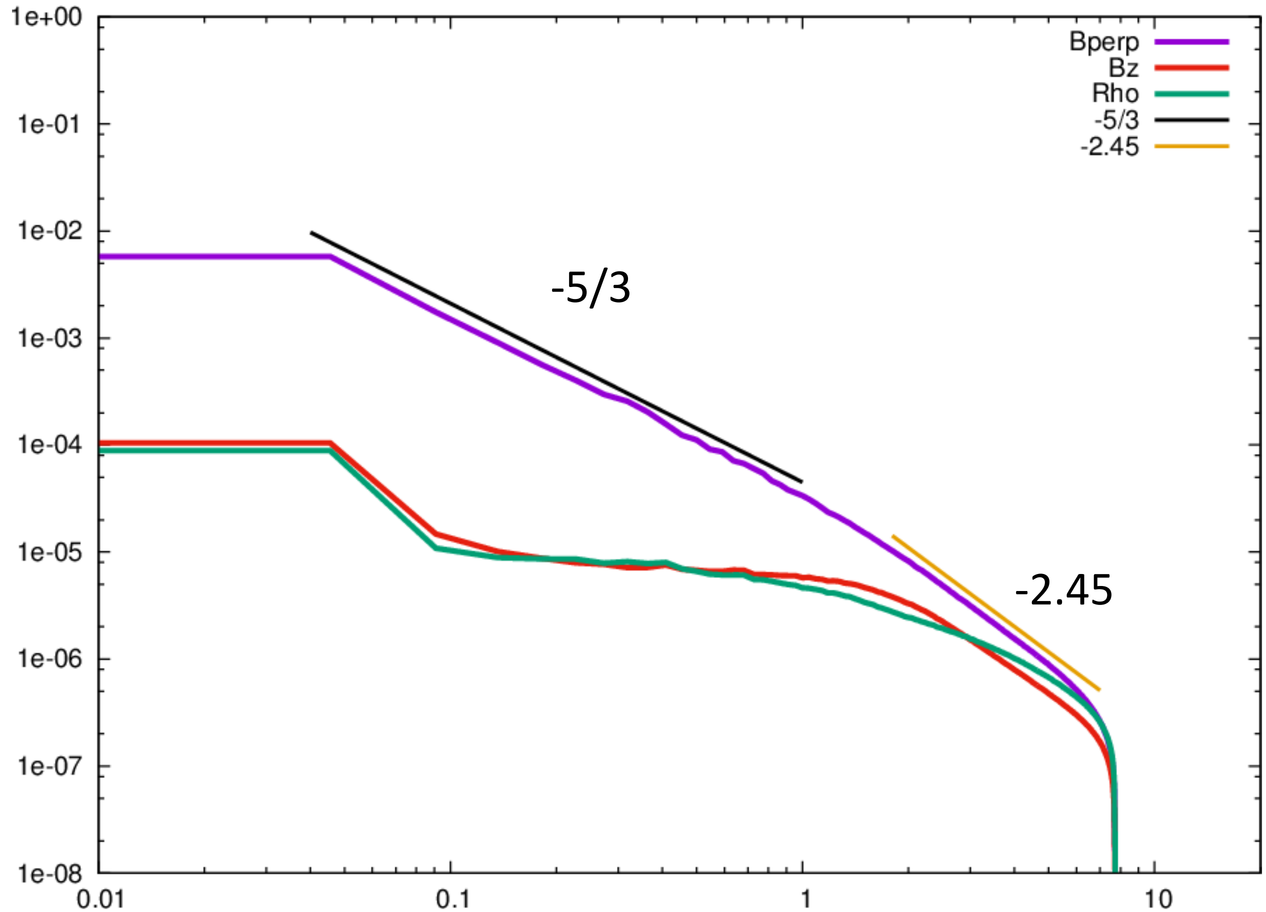
Initially, equal isotropic ion and electron temperatures with $\beta_i = 1$ (and also $\beta_i = 0.2$)

FLR-Landau fluid model is numerically integrated using a Fourier spectral method in a 3D periodic domain , 5.7 to 14 times more extended in the parallel direction than in the perpendicular ones, in order to focus on the quasi-transverse dynamics.

Weak hyperviscosity and hyperdiffusivity are supplemented
- to ensure the presence of a numerical dissipation range,
- to mimic the effect of Landau dissipation at ion scales not retained in the simulation

(do not affect spectral exponents) .

Resolution of 128^3 (up to 512^2x256) points before aliasing is removed.



Simulation at β=1 including a Kolmogorov range.
Clear spectral break near $k_\perp r_L = 1$
Flat density and B_z spectra at large scales that tend to asymptote the $B_\perp$ spectrum in the sub-ion range.

Simulations concentrating on the sub-ion range, performed for various amplitudes of turbulent Alfvenic fluctuations, and various propagation angles.

	Run A+	**Run A**	**Run B80**	**Run B83**	**Run B86**
Angle of injected KAWs	**80°**	**80°**	**80°**	**83.6°**	**86°**
rms of $v_\perp$ and $B_\perp$	**0.2**	**0.13**	**0.08**	**0.08**	**0.08**
$L_\perp/L_{//}$	0.18	0.18	0.18	0.11	0.07
rms of resulting density fluctuations	0.045	0.03	0.014	0.016	0.017
Transverse magnetic spectrum exponent	**-2.3**	**-2.6**	**-3.6**	**-2.8**	**-2.3**
$A = (k_z/k_0)(B_0/\delta B_{\perp 0})$	**0.9**	**1.4**	**2.2**	**1.4**	**0.9**

KAW modes driven at $|k\, d_i|$ =0.18 (the largest scales), and propagation angles with the ambient field of 80°, 83.6°and 86° (varied by changing the parallel size of the domain).

$\beta_i = \beta_e = 1$

A main result: the dynamics is strongly sensitive to the **nonlinearity parameter**

$$\boxed{\chi = \omega_{NL}/\omega_W}$$

ratio of the nonlinear frequency (of the transverse dynamics) to the kinetic Alfvén wave frequency (along the magnetic field lines)

$$\omega_{NL} = \sqrt{k_\perp^5 E(k_\perp)}$$ (constructed from electron velocity)

$$\omega_W = \overline{\omega}(k_\perp \rho_i) v_A k_{\|}$$

$\overline{\omega}(k_\perp \rho_i)$ given by linear kinetic theory

$k_{\|}$: wavenumber along the magnetic field lines (to be defined)

Turbulence anisotropy

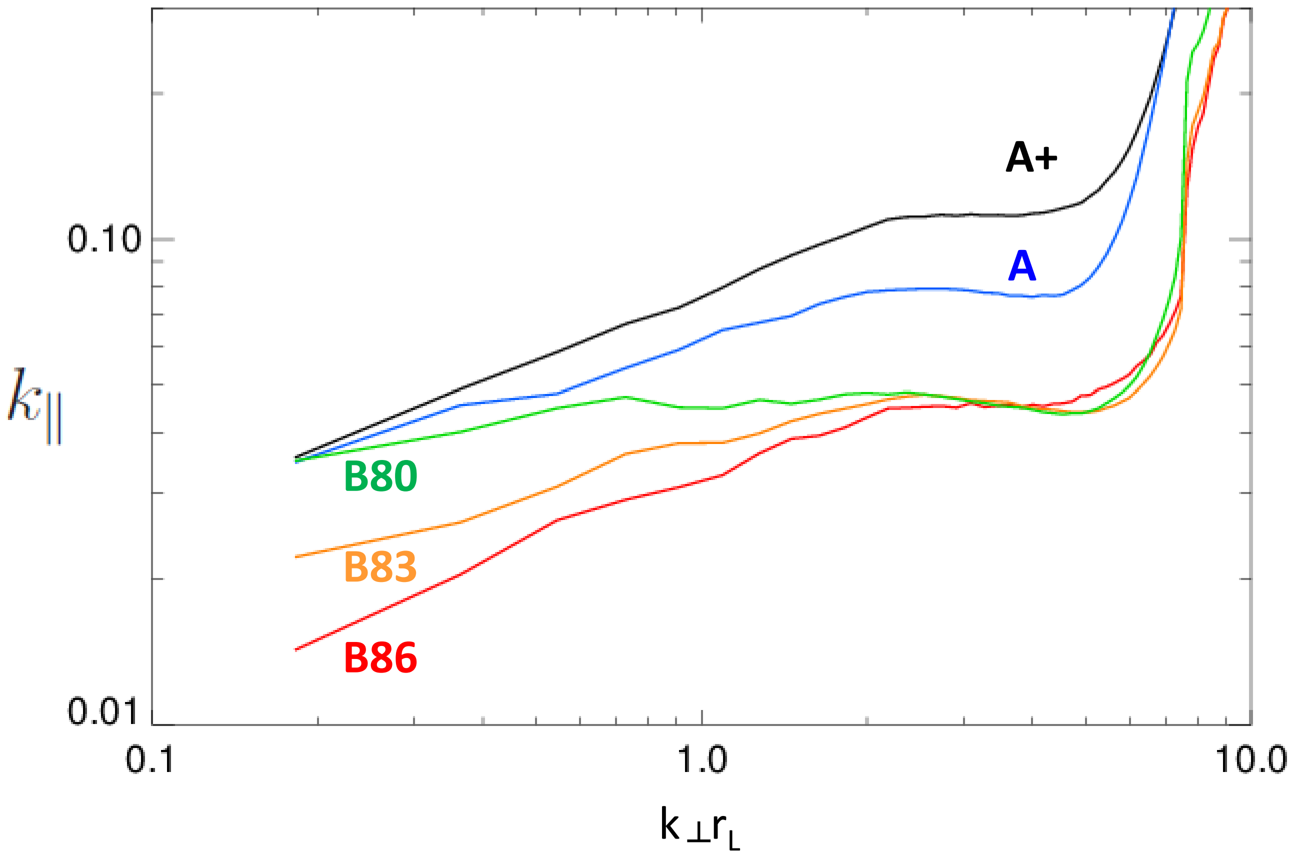
Parallel wave number along the local magnetic field line of an eddy with transverse wavenumber $k_\perp$ *(Chow & Lazarian, ApJL* ***615****, L41, 2004)*

$$k_\parallel(k_\perp) \approx \left(\frac{\sum_{k \le |\mathbf{k}'| < k+1} |\widehat{\mathbf{B}_L \cdot \nabla \mathbf{b}_l}|^2_{\mathbf{k}'}}{B_L^2 \sum_{k \le |\mathbf{k}'| < k+1} |\hat{\mathbf{b}}|^2_{\mathbf{k}'}} \right)^{1/2}$$

$\mathbf{B}_L$ is the local mean field obtained by eliminating modes whose perpendicular wavenumber is greater than $k/2$

The fluctuating field $\mathbf{b}_l$ is obtained by eliminating modes whose perpendicular wavenumber is less than $k/2$.

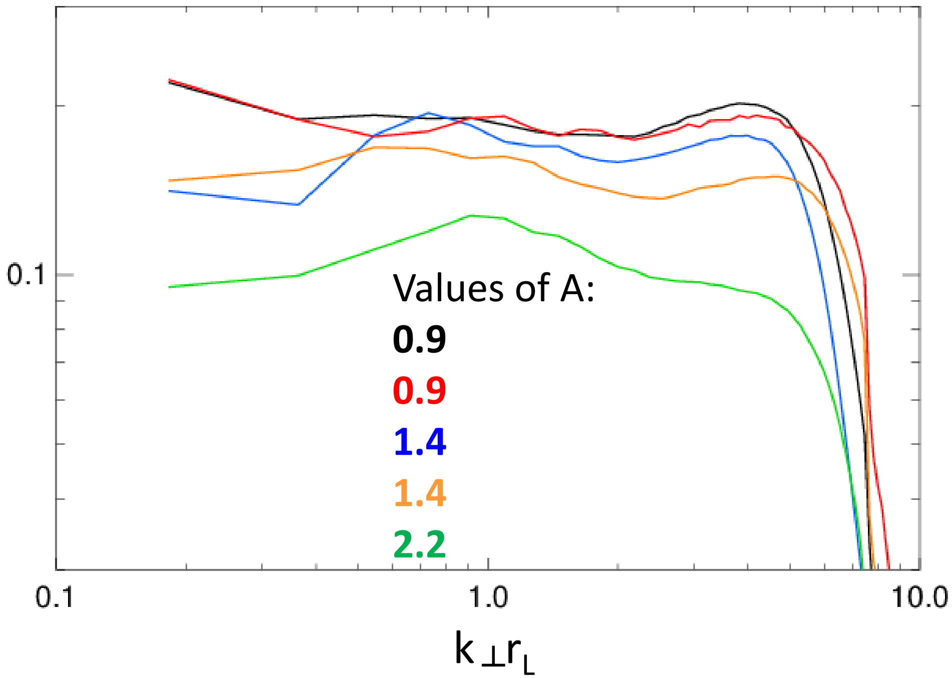
Parallel wavenumber defines the inverse correlation length ***along magnetic field lines***, at a specified transverse scale.



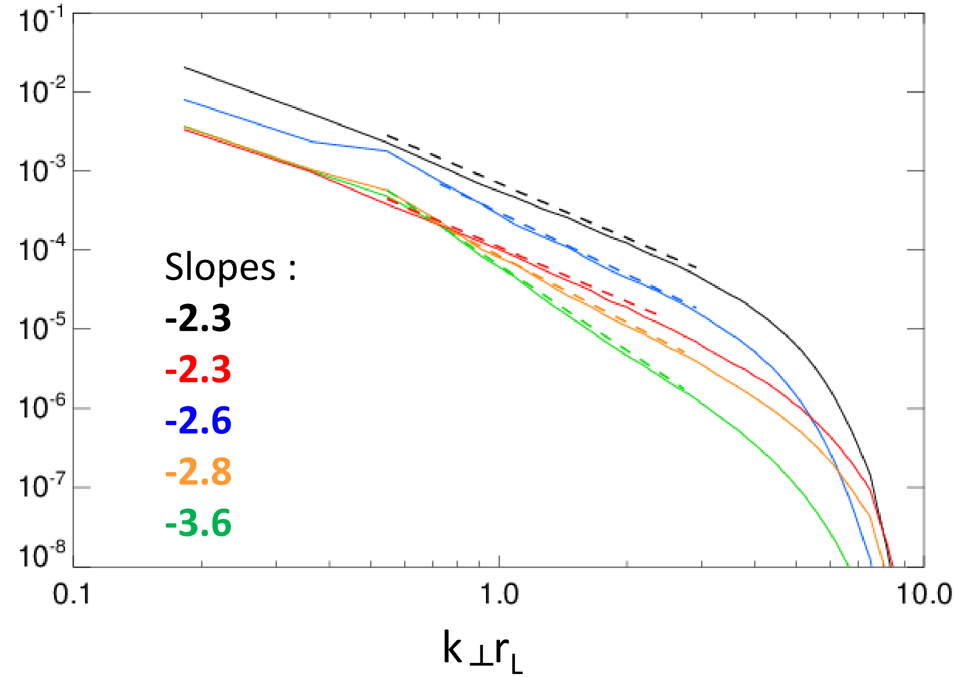
For small amplitude fluctuations, (B80), $k_{\|}$ is rather flat, suggesting weak turbulence.

For larger amplitudes, $k_{\|}$ grows as a power law (as expected in a strong turbulence regime), and saturates at small scales.

$\chi = \omega_{NL}/\omega_W$



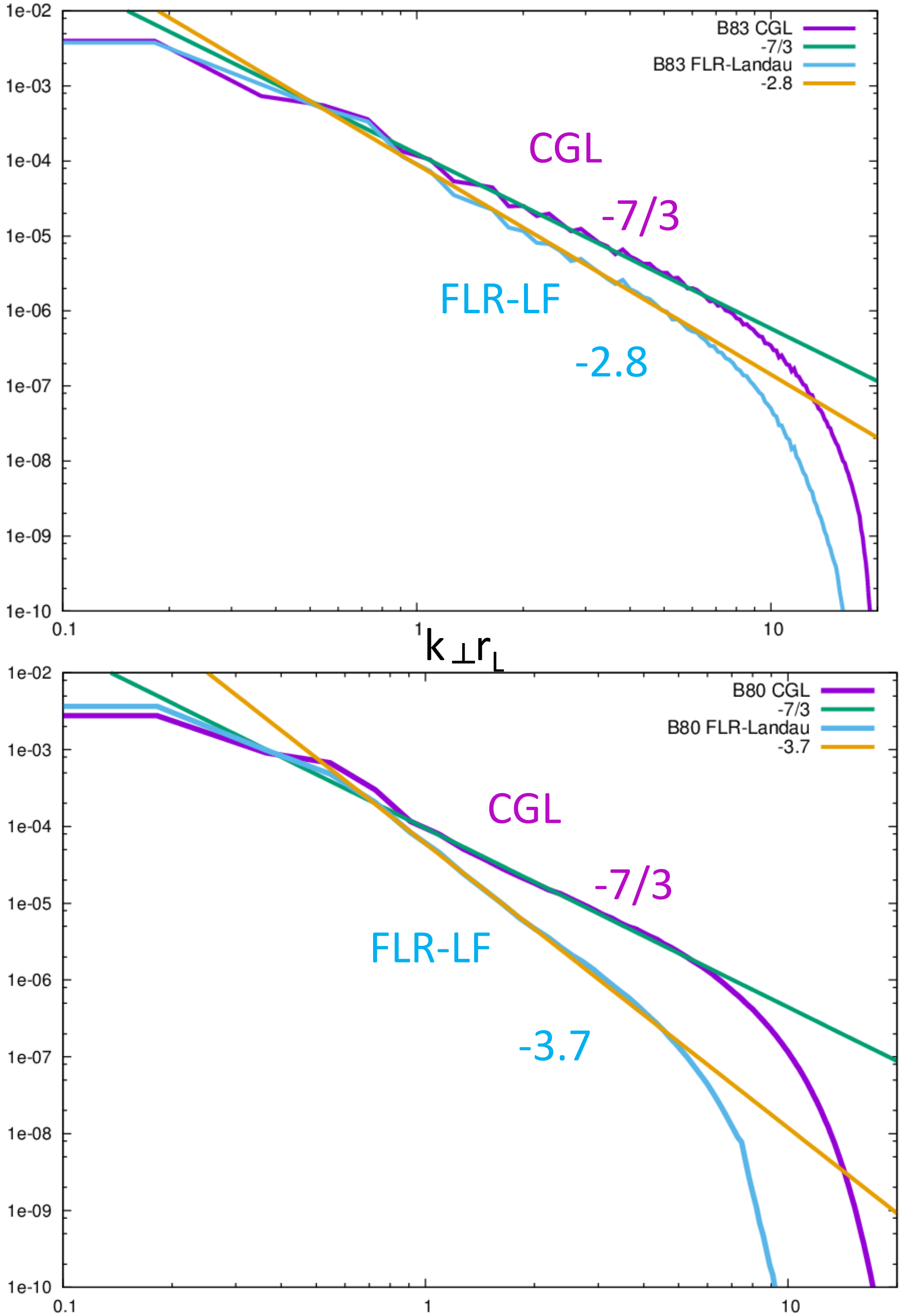
$E_{B\perp}(k_\perp)$



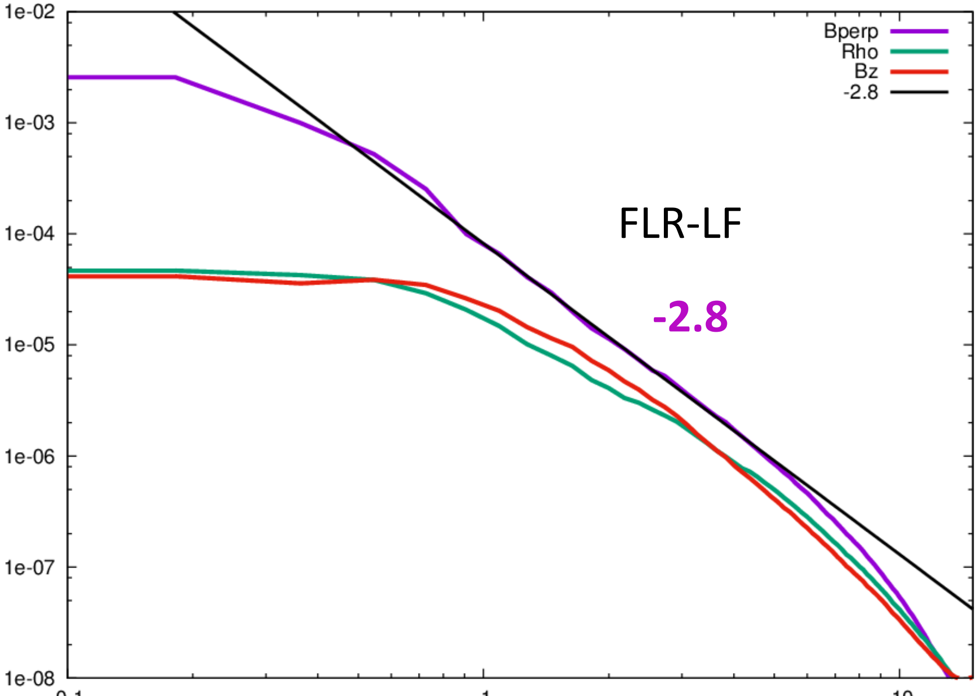
Spectra averaged over 150 Ω_i^{-1}
in the quasi-stationary regime.

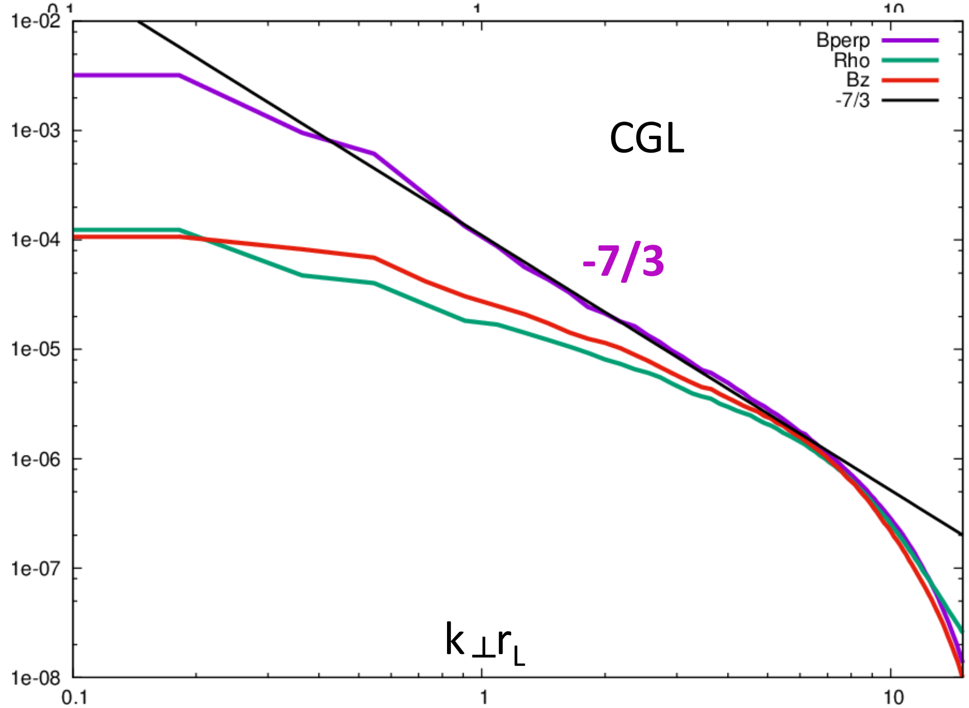
When the parameter $A = (k_z/k_0)(\delta B_{\perp 0}/B_0)^{(-1)}$ is small enough critical balance is satisfied.

Spectra are steeper when the nonlinearity parameter is smaller.



Magnetic spectra obtained with a **CGL** model with Hall effect (but **no Landau damping**), **display a -7/3 spectrum whatever the χ parameter.**





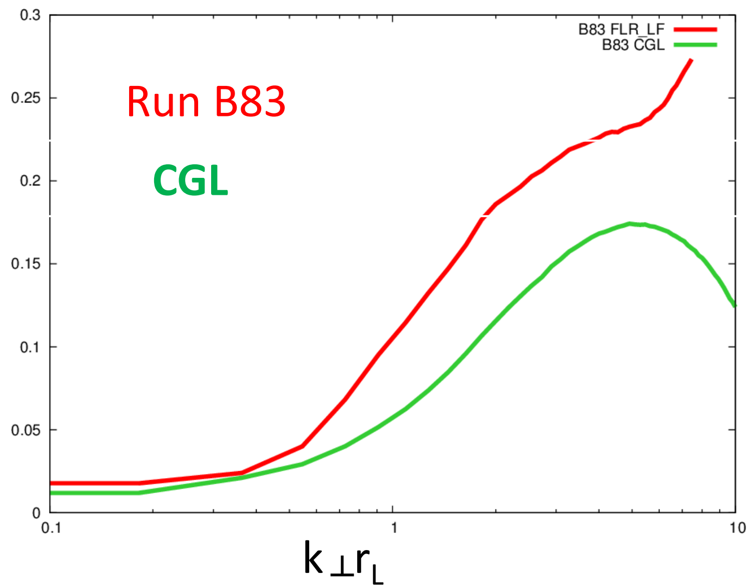
Density and B_z spectra more consistent with KAW linear theory with FLR-LF model.

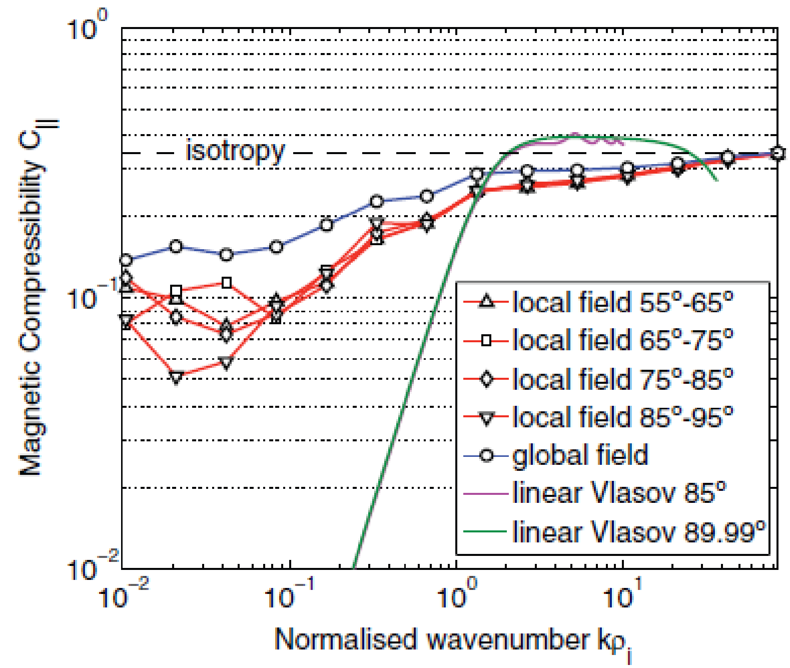
The flatter part at large scales due to selective damping of slow modes induced by Landau damping.

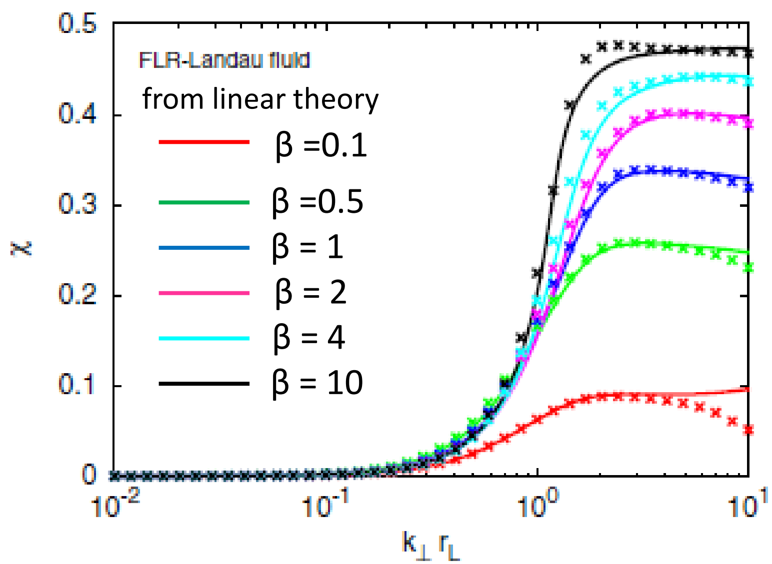
With CGL, no range where B_z and $B_\perp$ spectra are parallel.

Magnetic compressibility spectrum

$$\chi(k_\perp r_L) = |B_z(k_\perp r_L)|^2/|B(k_\perp r_L)|^2$$



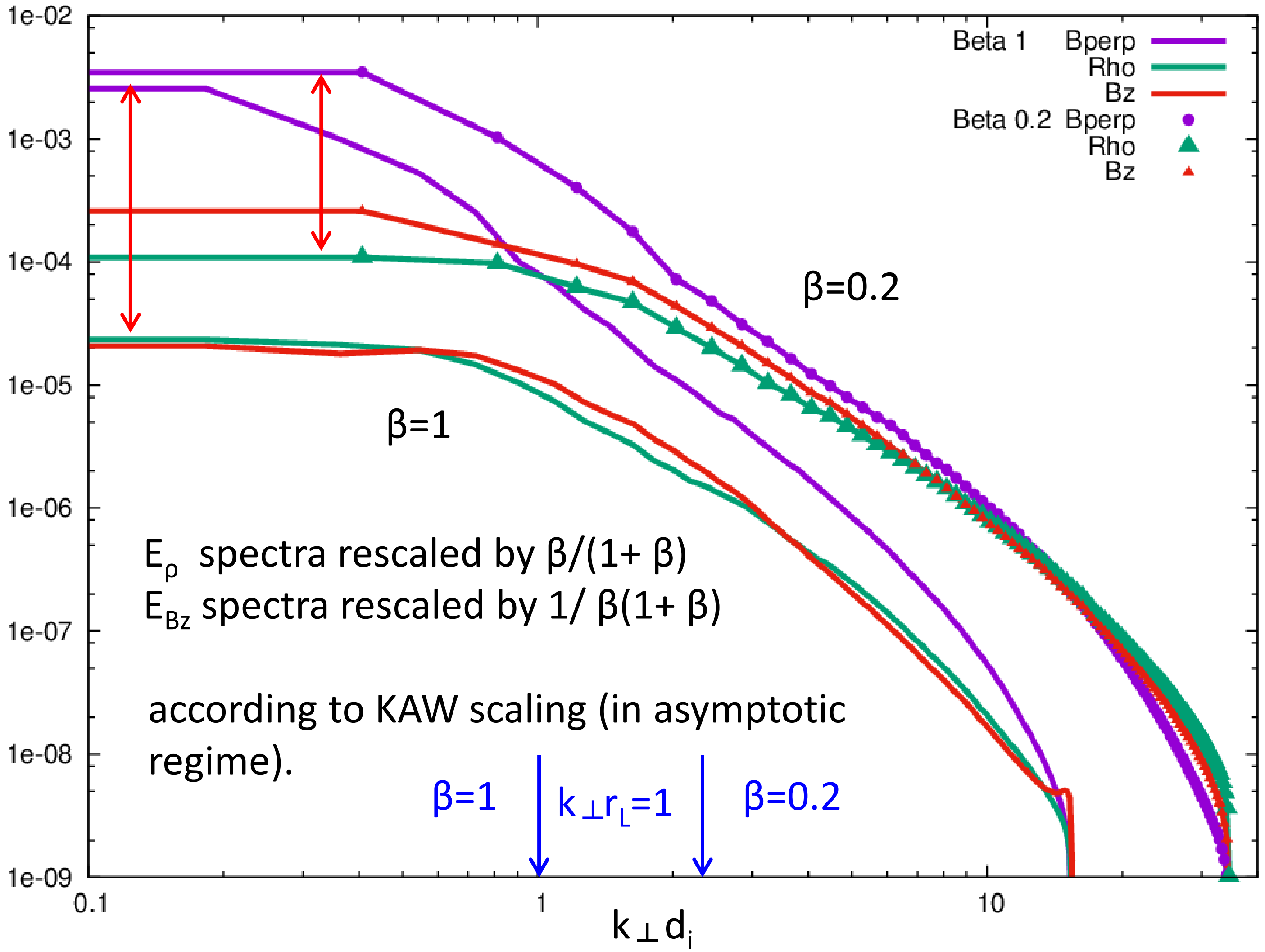




Magnetic compressibility from Cluster data
*(Kiyani et al. ApJ **763**, 10, 2013)*

Hunana, Golstein, Passot, Sulem, Laveder & Zank,
Astrophys. J. 766, 93 (2013); Solar Wind 13 Proceedings.

θ = 89.99° *(in order to accurately capture large $k_\perp$)*



No significant difference in the slope of $B_\perp$ spectrum for β=1 or β=0.2

Note a stronger damping of density and B_z component at β=1

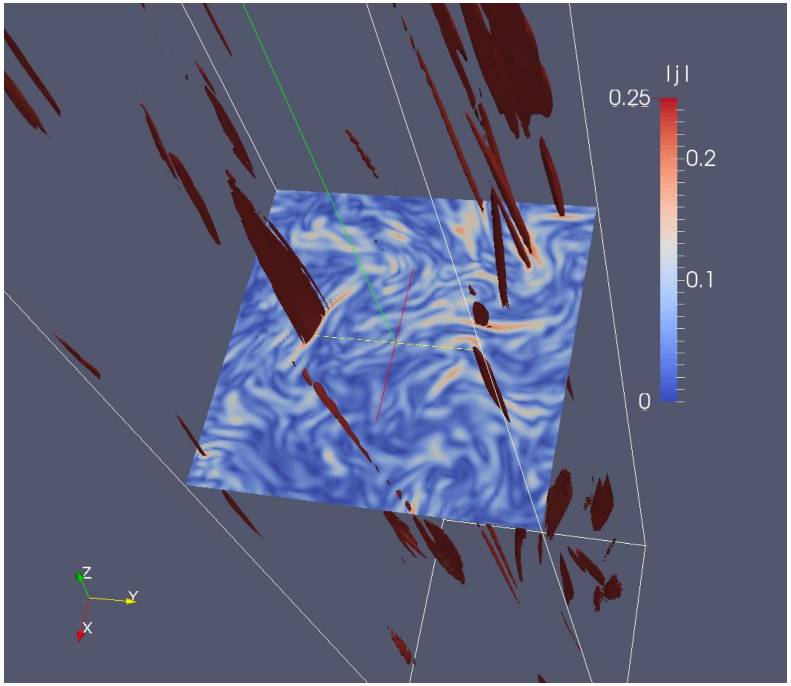
Structures of the electric current:

- Usual MHD leads to **current sheets**

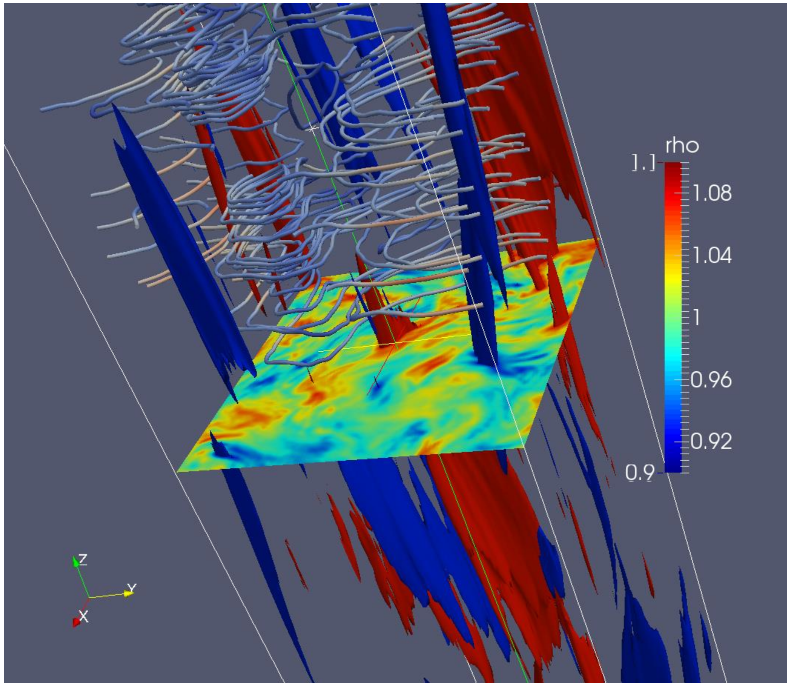
- **Current filaments** obtained in incompressible Hall-MHD
  *(Miura & Araki , J. Phys. Conf. Series **318**, 072032, 2011)*
  and in Electron MHD
  *(Meyrand & Galtier, Phys. Rev. Lett. **111**, 264501, 2013),*
  due to Hall term.

Both filaments and sheets are observed.

Run A



Current



Density and ion velocity field lines

Both current sheets and filaments.

A phenomenological model for KAW turbulence

Extend analysis of Howes et al. (2008, 2011) by

- Retaining the influence on the energy transfer time, of the process of **ion temperature homogenization along the magnetic field lines induced by Landau damping.**
- Improving description of nonlocal interactions.

Main results:

- Critical balance establishes gradually as $k_\perp$ increases, permitting a weak large-scale turbulence to become strong at small enough scales.
- Non-universal power-law spectrum for strong turbulence at the sub-ion scales with an exponent which depends on the saturation level of the nonlinearity parameter $\chi = \omega_{NL}/\omega_W$, covering a range of values consistent with solar wind and magnetosheath observations.

Stretching frequency Alfvén wave frequency

*T. Passot & P.L. Sulem, Astrophys. J. Lett., **812**, L37 (2015).*

For the sake of simplicity , concentrate on the case where nonlinear interactions are local, i.e. energy spectrum not too steep, which is the case for $\beta \approx 1$.
More general case addressed in Passot & Sulem (ApJL, 2015)

Involved frequencies:

Nonlinear frequency: $\omega_{NL} = \Lambda\, (k_\perp^5 d_i^2 E_k)^{1/2}$

Λ : numerical constant of order unity ; $E_k \equiv E(k_\perp)$

Frequency of KAWs propagating along the distorted magnetic field lines:

$$\omega_W = \overline{\omega} k_{||} v_A \text{ (where } \overline{\omega} \text{ scales like } k_\perp \rho_i)$$

KAW Landau damping rate: $\gamma = \overline{\gamma} k_{||} v_A$

(where $\overline{\gamma}$ scales approximately like $k_\perp^2 \rho_i^2$, at least for β of order unity)

Homogenization frequency (for each particle species) : $\boxed{\omega_H = \mu k_{||} v_{th}}$

(where μ is a proportionality constant of order unity)

In the case of ions, comparable to other inverse characteristic time scales.

The corresponding frequency is much higher in the case of electrons (due to mass ratio), making electron homogenization along magnetic field lines too fast to have a significant dynamical effect.

Determination of the inverse transfer time or its inverse ω_{tr}

Proceeding as in the spirit of the two-point closures for hydrodynamic (Orzsag 1970, Sulem et al. 1975, Lesieur 2008) or MHD (Pouquet 1976) homogeneous turbulence,

$$\omega_{tr} = \frac{\omega_{NL}^2}{\omega_W + \omega_H} = \frac{\Lambda^2 \overline{\alpha}^2 k_\perp^3 E_k}{\overline{\omega} v_A k_\| + \mu v_{th} k_\|}$$

where $\overline{\alpha} = \overline{\omega} \sim k_\perp \rho_i$.

Turbulence energy flux: $\epsilon = C\omega_{tr} k_\perp E_k$

where *C* is a negative power of the Kolmogorov constant.

It follows that

$$\boxed{\epsilon = \frac{C\Lambda^2 \overline{\alpha}^2 k_\perp^4 E_k^2}{(\overline{\omega} v_A + \mu v_{th}) k_\|}}$$

where the homogenization frequency contribution becomes negligible at scales for which $k_\perp \rho_i \gg 1$.

Assuming a critically balanced regime where $k_{\|} v_A = (k_\perp^3 E_k)^{1/2}$,

one has $\omega_{NL} = \Lambda\, \omega_W$

leading to identify the constant Λ with the nonlinearity parameter.

One thus gets

$$\boxed{E_k \sim \Lambda^{-4/3} C^{-2/3} \epsilon^{2/3} k_\perp^{-7/3}.}$$

Here, due to Landau damping, **ε** is a function of $k_\perp \rho_i$ and decays along the cascade.

Phenomenological equation for KAW's energy spectrum when retaining linear Landau damping *(Howes et al. 2008, 2011)*

Steady state, outside the Injection range

$$\cancel{\partial_t E_k} + \underbrace{\mathcal{T}_k}_{\text{transfer}} = \underbrace{-2\gamma E_k}_{\text{Landau damping}} + \underbrace{\cancel{S_k}}_{\text{driving term acting at large scales}}$$

$$\mathcal{T}_k = \partial\epsilon / \partial k_\perp$$

In a critically balanced regime, this equation is solved as

$$\epsilon = \epsilon_0 \exp\left[-2C^{-1}\Lambda^{-2}\int_{k_0}^{k_\perp} \frac{\overline{\gamma}}{\xi\overline{\alpha}^2}(\overline{\omega} + \mu\beta^{1/2})d\xi\right]$$

From the linear kinetic theory,

$$\begin{cases} \overline{\alpha} = \overline{\omega} \\ \overline{\gamma}/\overline{\omega}^2 \approx \delta(\beta) \approx 0.78\rho_e/\rho_i \quad \text{when } \beta = 1 \\ \overline{\omega} \approx a(\beta)k_\perp \text{ with } a(\beta) = (1+\beta)^{-\frac{1}{2}}. \end{cases}$$

This leads to $\epsilon_k \sim \epsilon_0 k_\perp^{-\zeta} \exp[-2a(\beta)C^{-2}\Lambda^{-2}\delta(\beta)k_\perp]$

with $\boxed{\zeta = 2\delta(\beta)C^{-1}\mu\Lambda^{-2}\beta^{1/2}.}$

Involves proportionality constants C and μ which are to be empirically determined by prescribing for example that the exponential decay occurs at the electron scale.

Finally,

$$\boxed{E_k \sim k_\perp^{-(7/3+2\zeta/3)} \exp\left[-\frac{4}{3}a(\beta)\delta(\beta)C^{-1}\Lambda^{-2}(k_\perp\rho_i)\right].}$$

The correction in the exponent is not universal: expected when dissipation and nonlinear transfer times display the same wavenumber dependence (Bratanov et al. PRL **111**, 075001 (2013)).

More quantitative analysis by numerical simulations of the differential system.

For example:

$$\textbf{rate of strain: } \omega_{NL} \sim k_\perp v_{e\,k} = \Lambda \sqrt{\int_0^{k_\perp} \overline{\alpha}^2 p_\perp^2 E_p dp}$$

(by electron velocity gradients)

Transverse magnetic spectrum

$$\overline{\alpha} = 1 \text{ for } k_\perp \rho_i \ll 1 \qquad (v_{e\,k} \approx v_{i\,k})$$

$$\overline{\alpha} \sim k_\perp \rho \text{ for } k_\perp \rho_i \gg 1 \qquad (v_{e\,k} \sim j)$$

rate of strain due to all the scales larger than $1/k_\perp$ *(Elisson 1961, Panchev 1971)*

Local expression recovered when the Integral diverges at large $k_\perp$

is replaced by:

$$d\omega_{\rm NL}^2/dk_\perp = \Lambda^2 \beta^{-1} \overline{\alpha}^2 k_\perp^2 E_k$$

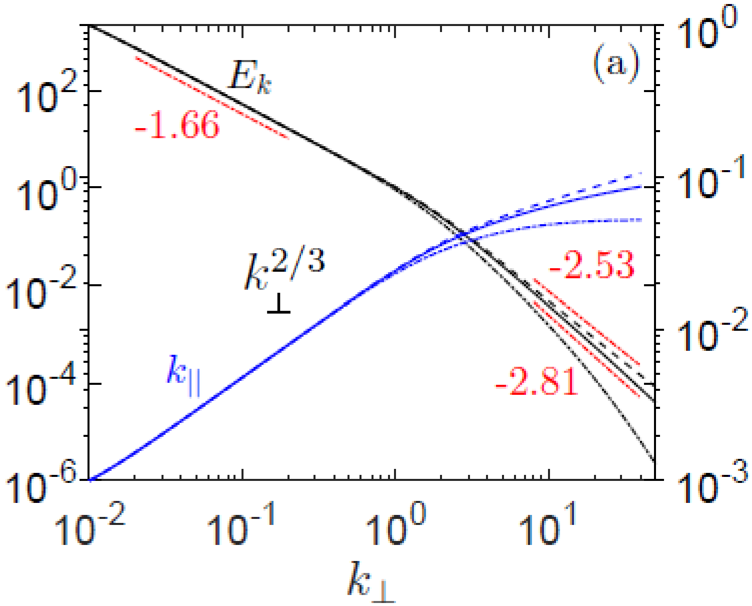
Differential system:

- Retains nonlocal interactions (relevant for relatively steep power-law spectra)
- Permits variation of the nonlinear parameter along the cascade and transition from large-scale weak turbulence to small-scale strong turbulence

The functions γ and ω are obtained using the WHAMP software

Phenomenological model

β=1



$\Lambda = 2$ (dashed lines), 1 (solid lines) and 0.5 (dotted lines)

Range for extended sub-ion power law

Λ	0.71	1	1.22	1.30	1.41	2	4.47
exponent	-3.18	-2.81	-2.68	-2.66	-2.63	-2.53	-2.45

Sub-ion exponent depends on the saturation value Λ of the nonlinear parameter. Range of variations comparable to observations.

Solar wind observations

Correlations were made between slope in transition range and power in the inertial range: higher power leads to steeper spectrum

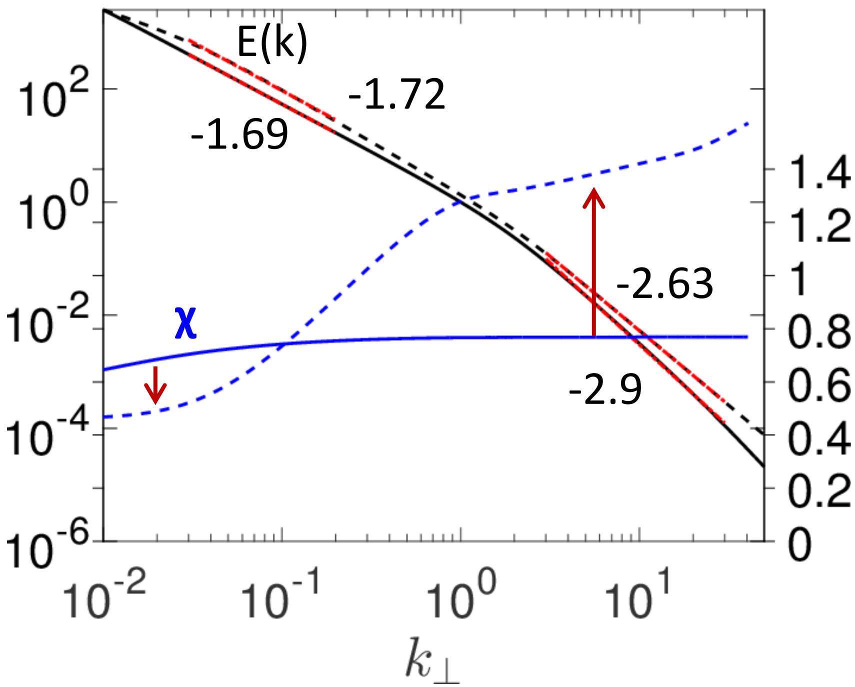
Bruno et al. ApJL **739** L14 (2014)

Present work

Two comments are in order:

1. does not consider a transition range
2. the correlation is made with the nonlinearity parameter in the sub-ion range

When χ is not strictly constant, it can happen that the branch with the higher χ in the sub-ion range corresponds to that with the smaller one in the Kolmogorov range.



Conclusion

Main features of the phenomenological model:

- Introduction of a new time scale associated with the homogenization process along magnetic field lines, induced by Landau damping

- The model predicts a non-universal power-law spectrum for strong turbulence at the sub-ion scales with an exponent which

  - depends on the saturation level of the nonlinearity parameter,
  - covers a range of values consistent with solar wind and magnetosheath observations.

3D FLR-Landau fluid simulations of Alfvenic turbulence at the ion scales

- Spectral index is not universal
  (varied by changing amplitude and angle of driven KAWs).

- Critical balance is satisfied when fluctuations are strong enough.

- Influence of Kolmogorov range on sub-ion range and of β still to be analyzed.