Vlasov simulation study of electron acceleration by large amplitude electron Bernstein waves

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Bengt Eliasson

ABP Group, Physics Department, SUPA
Strathclyde University, UK

Collaborators:
D. C. Speirs, Strathclyde University, UK
K. Papadopoulos, A. Najmi, G. Milikh, X. Shao, U. Maryland
L. Daldorff, NASA Goddard Space Flight Center & Catholic U. of America, DC

Outline

A. Artificial aurora and descending ionospheric fronts in recent heating experiments

B. High-frequency turbulence induced by large amplitude electromagnetic waves

C. Anomalous absorption on striations

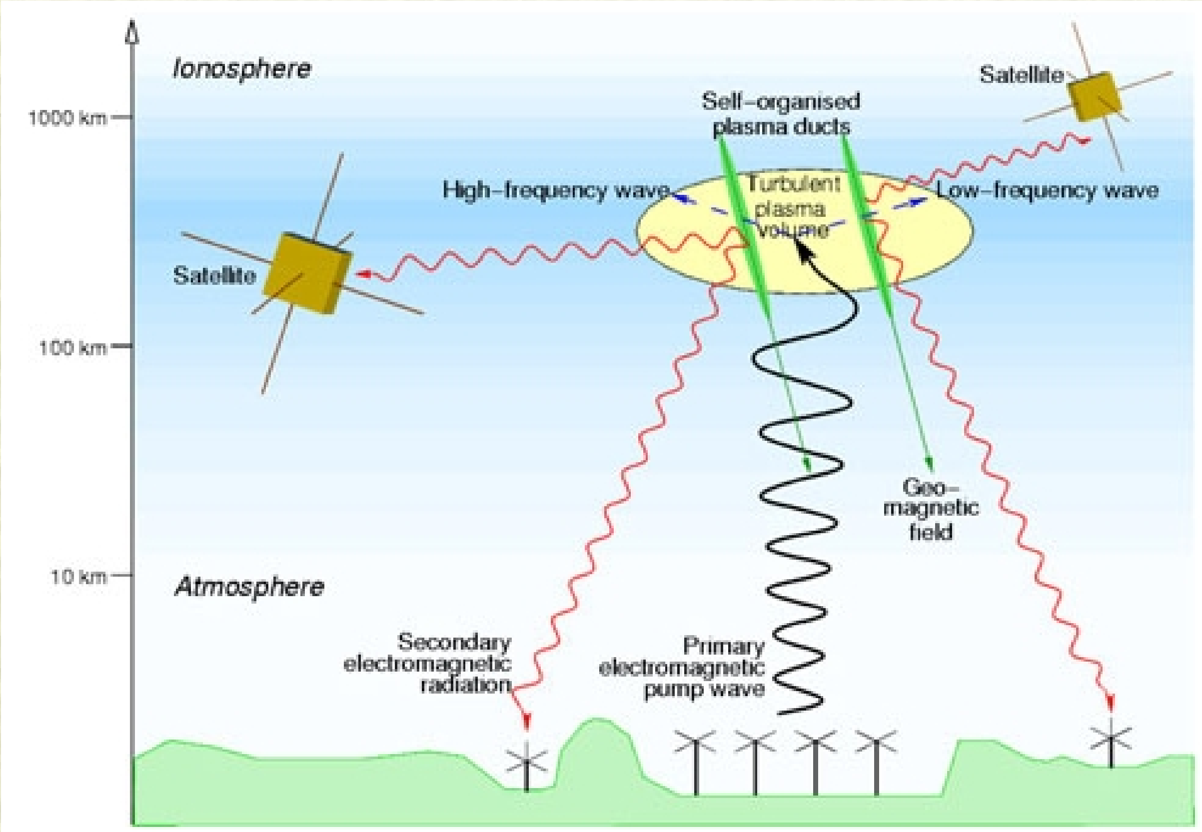
D. Stochastic electron heating by large amplitude electron Bernstein waves

E. Electron acceleration by strong Langmuir turbulence, ionization of neutral gas

F. Summary

Sketch of experimental setup

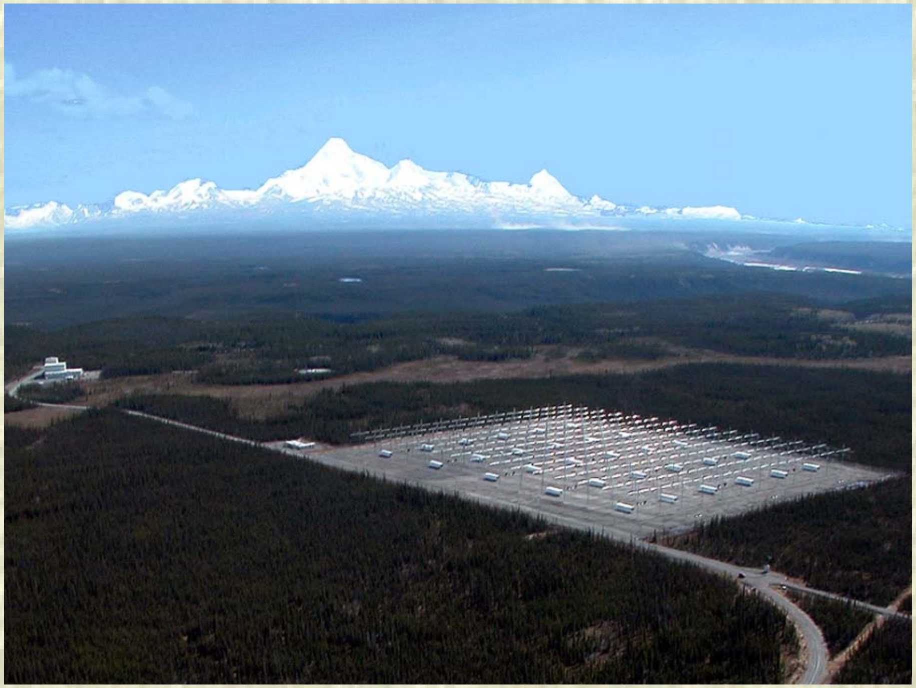
The Earth's ionosphere used as a natural laboratory to study turbulence in an unlimited magnetised plasma.



Diagnostics: Escaping radiation, radars, optical emissions, etc.

Courtesy of Bo Thidé (www.physics.irfu.se)

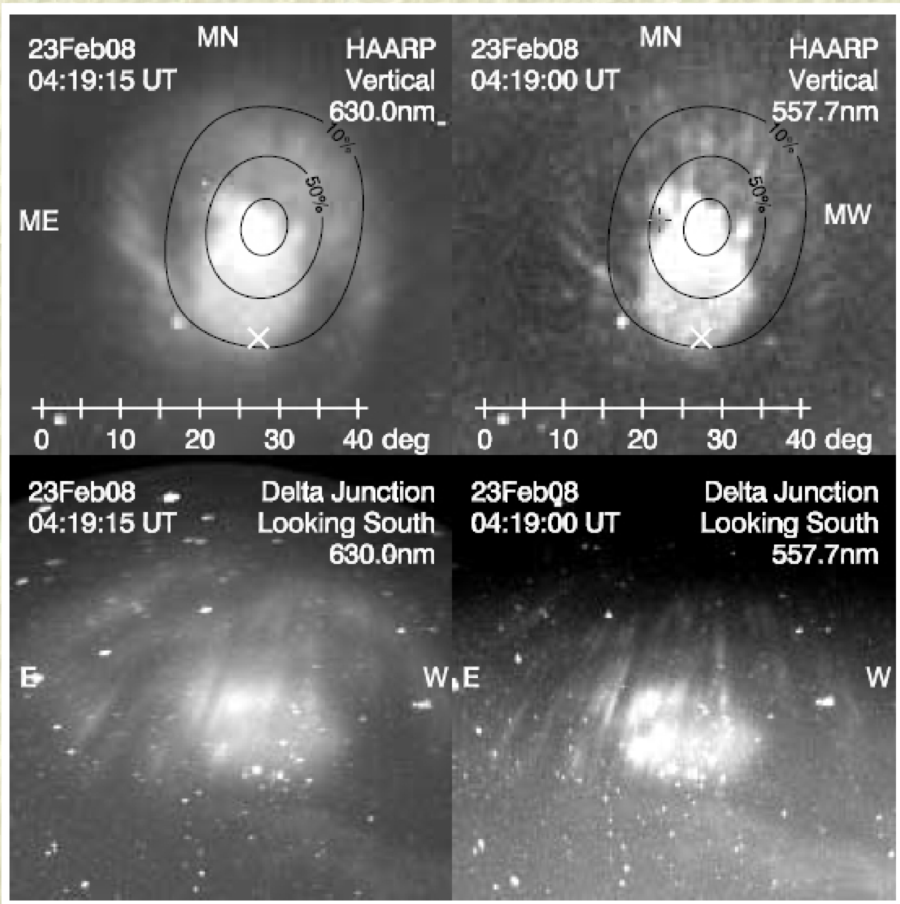
High Frequency Active Auroral Research Program (HAARP)

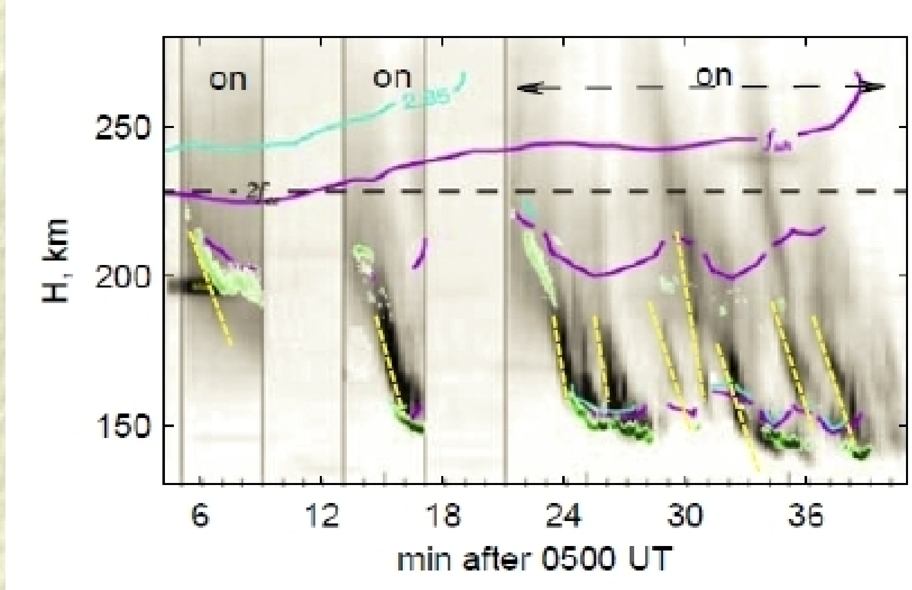


HAARP research station, near Gakona, Alaska

Established 1993, last major upgrade 2007.

Observations of descending aurora above HAARP



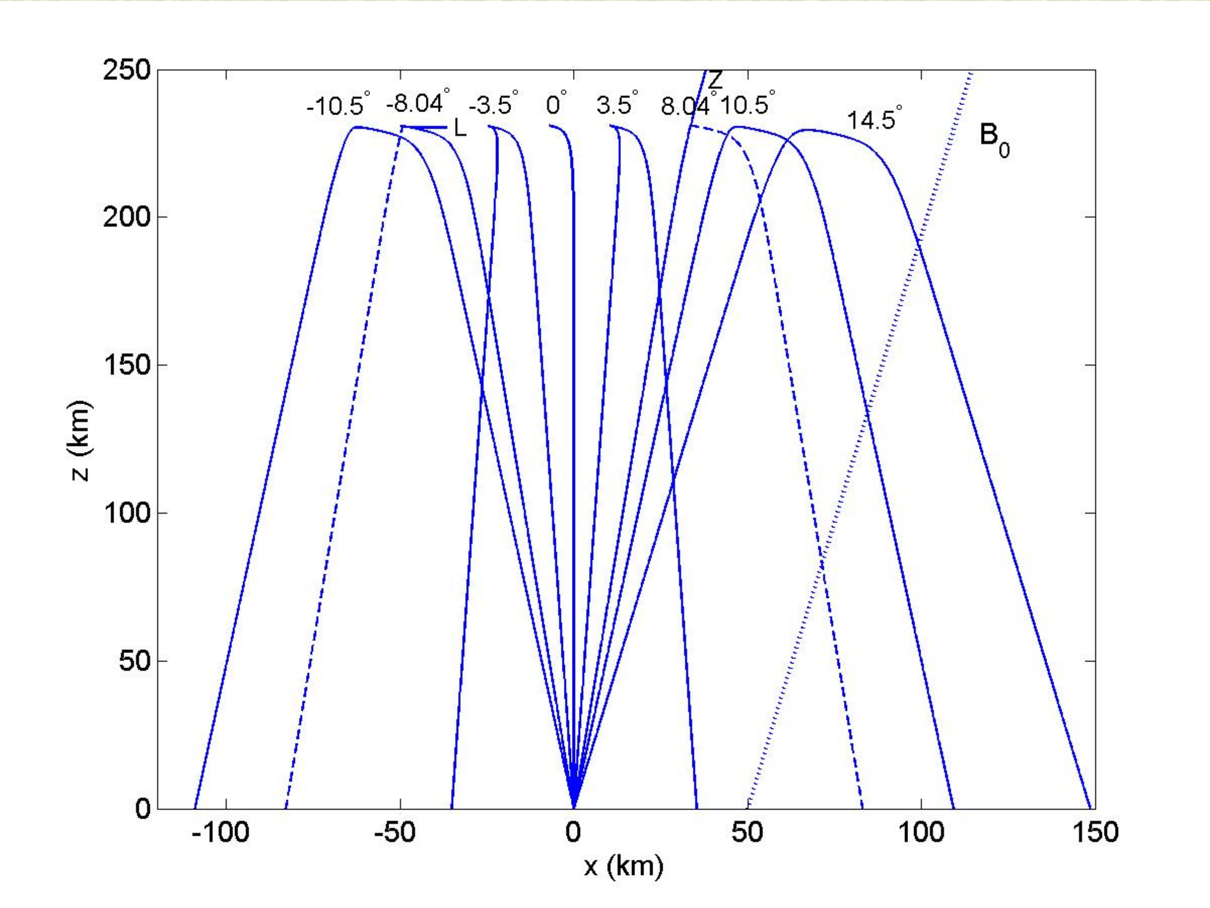


Time-vs-altitude plot of **557.7** nm optical emissions along *B* with contours showing the altitudes where *fp =2.85 MHz (blue)*, *UHR= 2.85* MHz (violet), and $2f_{ce}$ = *2.85 MHz (dashed white)*. *Horizontal blips* are stars. **Shown in green is the Ion Acoustic Line intensity.**

✓**the artificial plasma near h_{min} was quenched several times.**

Pedersen, Gustavsson, Mishin *et al.*, Geophys. Res. Lett., 36, L18107 (2009).
Pedersen, Mishin *et al.*, Geophys. Res. Lett., 37, L02106 (2010).
Mishin & Pedersen, Geophys. Res. Lett., 38, L01105 (2011).

Rays of ordinary (O) mode waves



Ray-tracing

$\frac{d\mathbf{k}}{dt} = -\nabla_{\mathbf{r}}\omega$

$\frac{d\mathbf{r}}{dt} = \nabla_{\mathbf{k}}\omega$

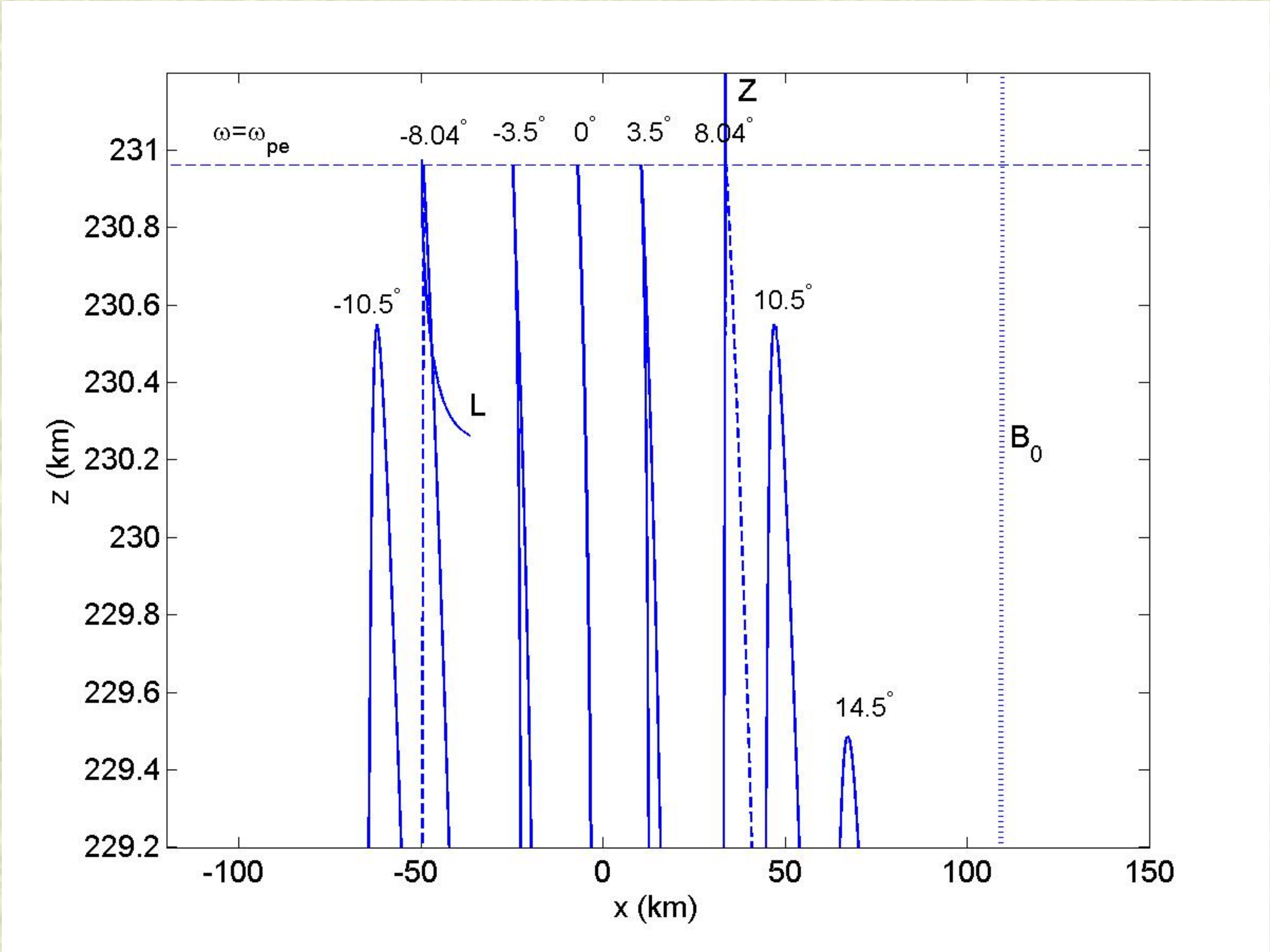
Appleton-Hartree dispersion relation gives $\omega(\mathbf{k}, \mathbf{r})$

Magnetic field $\mathbf{B}_0 = 5 \times 10^{-5}\,\mathrm{T}$, tilted $\theta = 14.5°$ to vertical. Electron cyclotron frequency $f_{ce} = 1.4\,\mathrm{MHz}$.

$f_0 = 3.2$ MHz transmitted frequency, ~ 100 m vacuum wavelength.

Ordinary mode waves are reflected near the critical layer where $\omega = \omega_{pe}$.

Rays closeup near reflection point

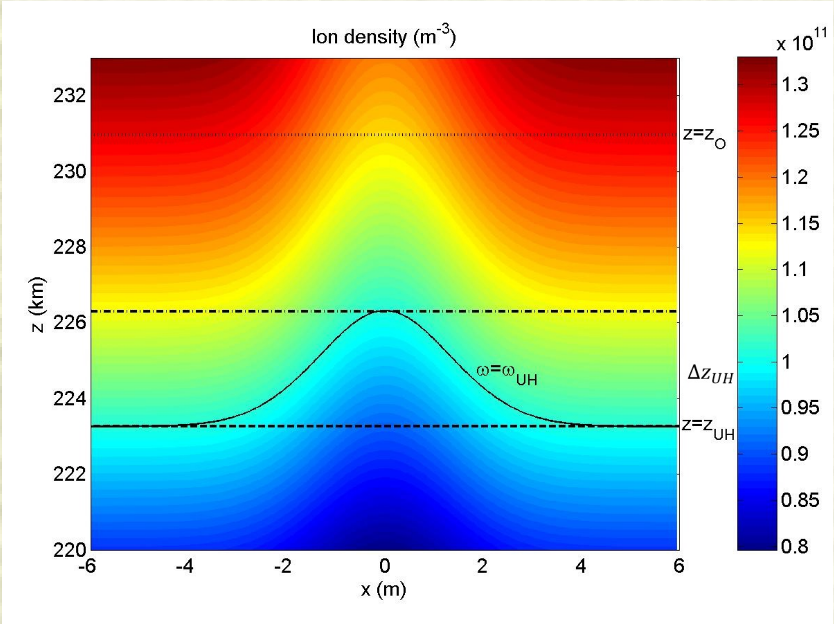


Rays within the Spitze region $\chi_S = \pm\arcsin[\sqrt{Y/(1+Y)}\sin(\theta)] \approx \pm 8.04^\circ$ reach the critical layer.

Anomalous absorption of electromagnetic waves

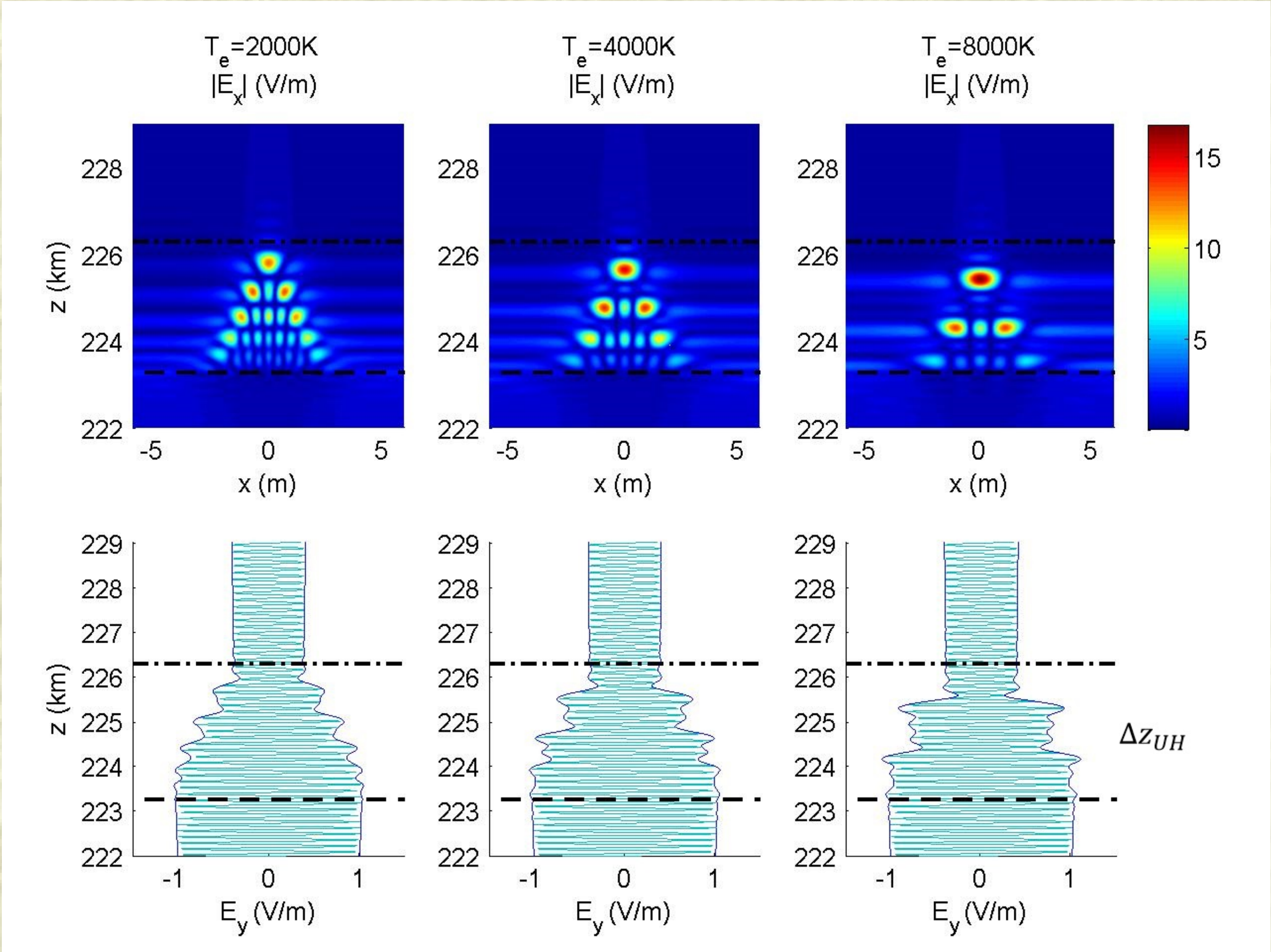
- It is observed that O mode radio waves injected along the magnetic field lines become absorbed by the ionosphere after about one second of heating
- Happens when the transmitted frequency is below the maximum upper hybrid frequency of the ionosphere
- Believed to be due to mode conversion to upper hybrid waves on density striations created due to thermal instability

Conversion O mode to upper hybrid waves

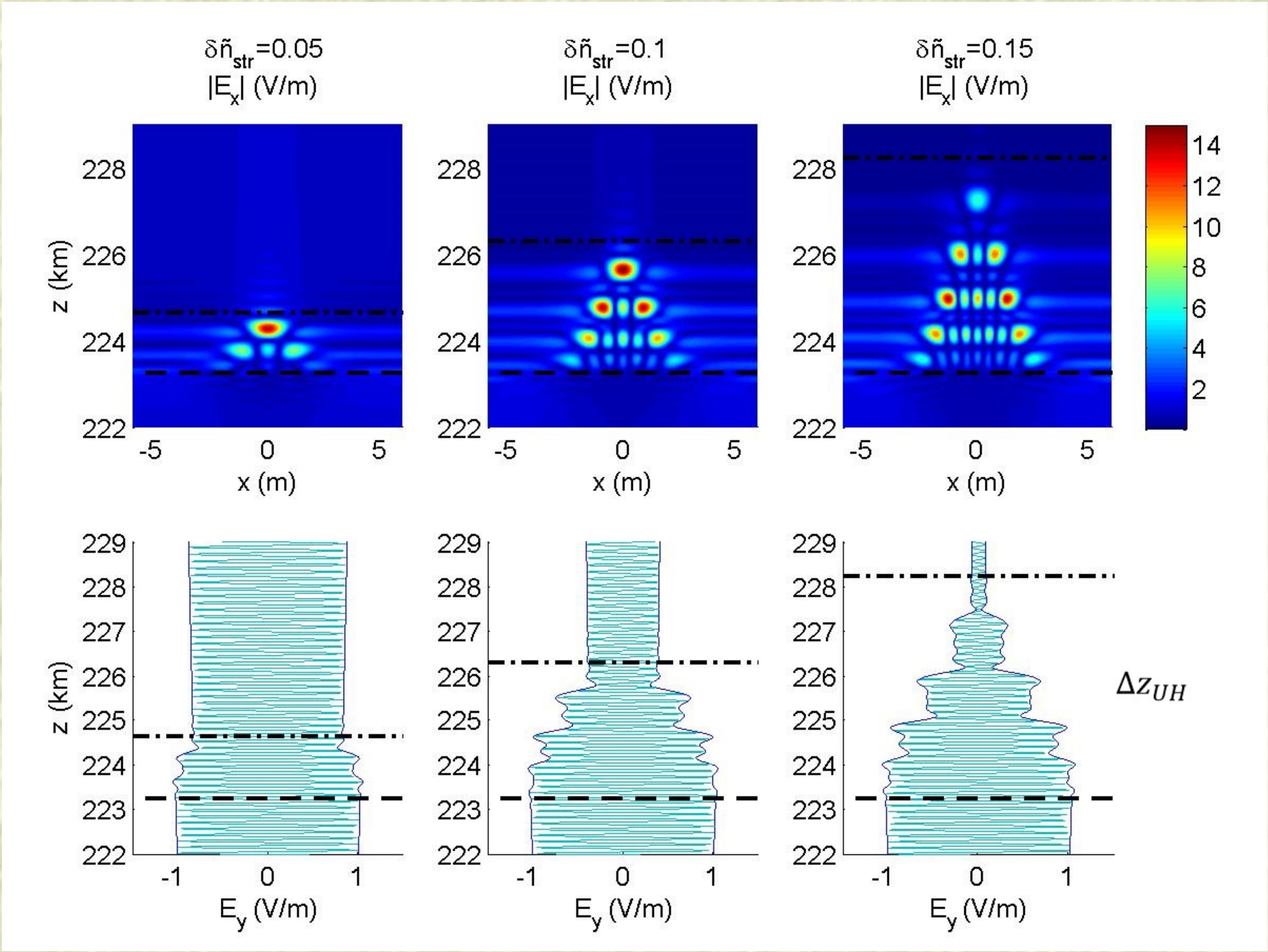


- O mode waves reflected at critical altitude $z = z_O$ where $\omega = \omega_{pe}$.
- Solid lines: Where locally $\omega = \omega_{UH}$ mode conversion O mode to upper hybrid waves can take place.
- Full-wave simulations to study the coupling between O mode and UH waves. Coordinate system such that z-axis along the magnetic field.

Eliasson & Papadopoulos, Geophys. Res. Lett. **42**, 2603 (2015).



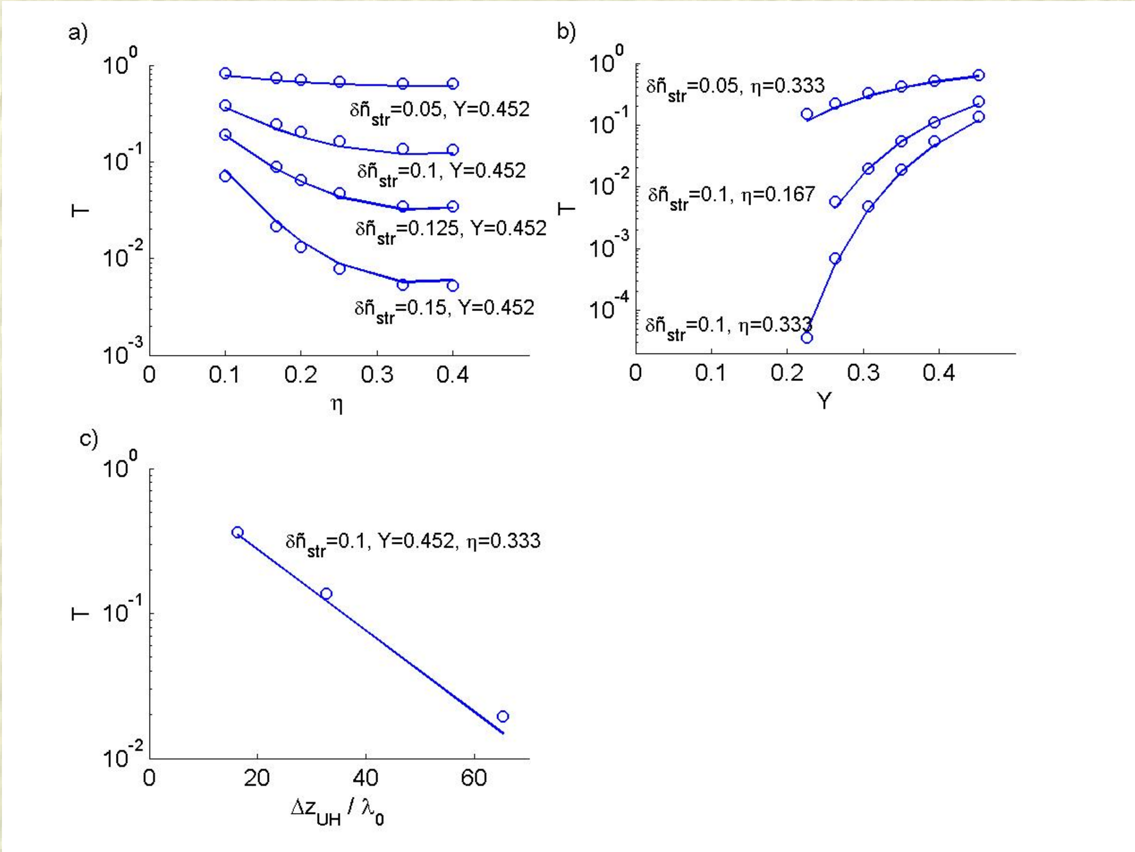
Excitations of UH waves (top) at quantized heights where UH frequency matches resonance frequency (Sturm-Liouville problem). Absorption of O mode wave (bottom) not dependent on electron temperature.



Absorption strongly dependent on striation depth.

Increased absorption with decreasing magnetic field and with increasing plasma length-scale and density of striations.

Expression for transmission coefficient

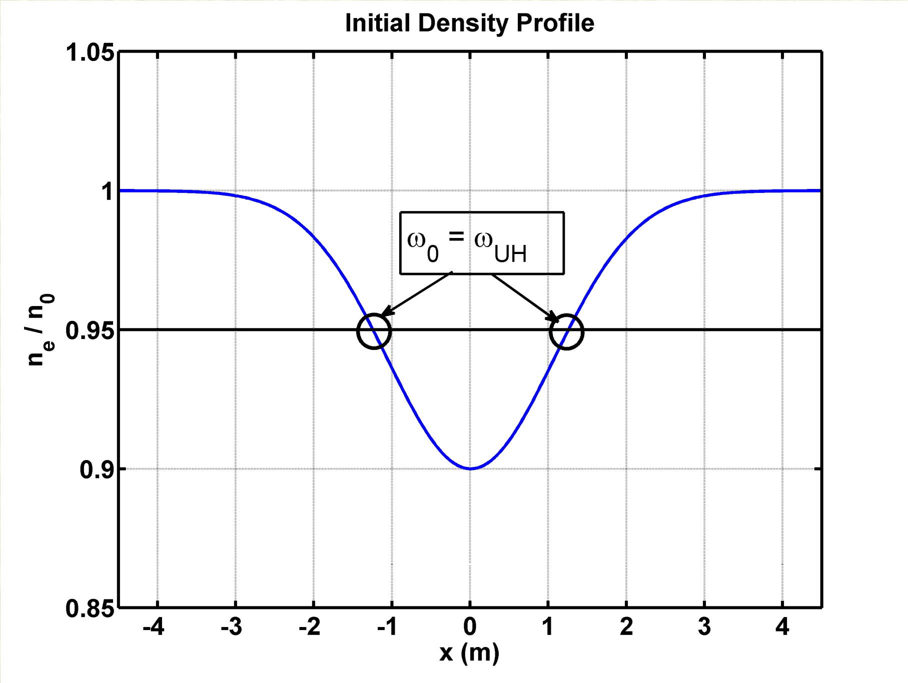


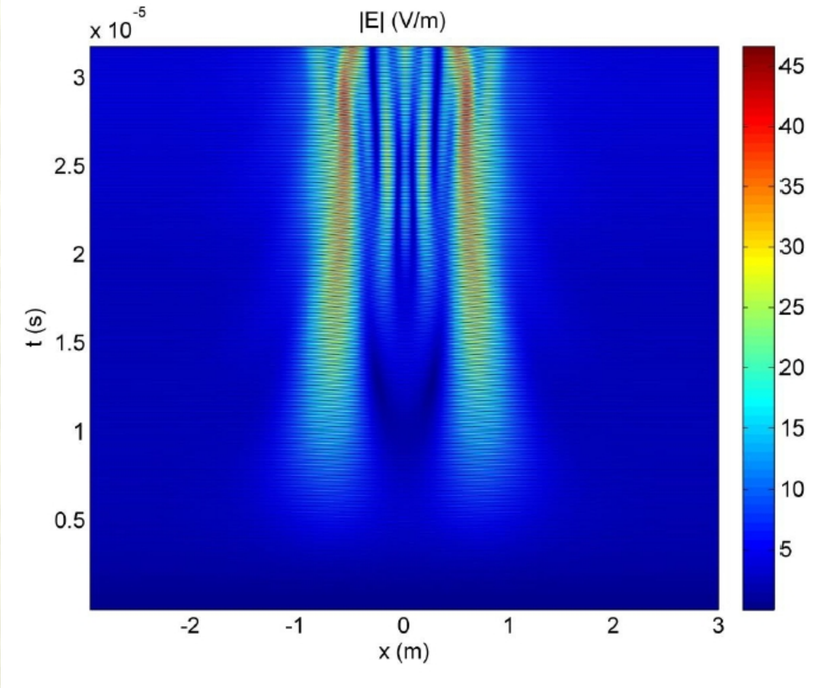
Comparison simulation results (circles) and numerical fit to expression

$$T = \exp\left[-3.24\delta\tilde{n}_{str}\frac{\Delta z_{UH}}{\lambda_0}(\eta - 1.4\eta^2)(\frac{1}{Y} - 1.09)\right], \quad Y = \frac{\omega_{ce}}{\omega_0}$$

Eliasson & Papadopoulos, Geophys. Res. Lett. **42**, 2603 (2015).

Vlasov simulations: Mode conversion to UH waves





$$\frac{\partial f_\alpha}{\partial t} + v_x \frac{\partial f_\alpha}{\partial x} + \frac{q_\alpha}{m_\alpha}(\widehat{\mathbf{x}}(E + E_{ext}) + \mathbf{v} \times B_0 \widehat{\mathbf{z}}) \cdot \nabla f_\alpha = 0$$

$$\frac{\partial E}{\partial x} = \frac{e}{\epsilon_0} \int (f_i - f_e)\, d^2 v$$

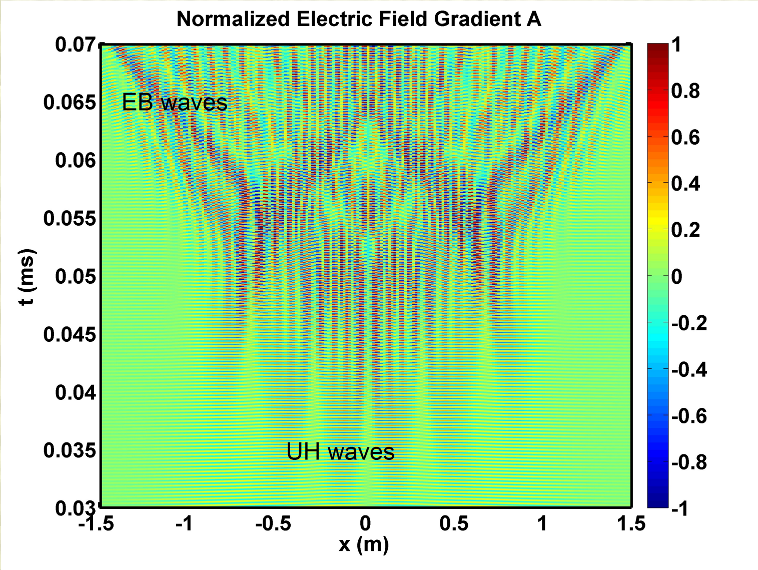
$E_{ext} = E_0 \sin(\omega_0 t)$, Dipole oscillating field representing the O mode.

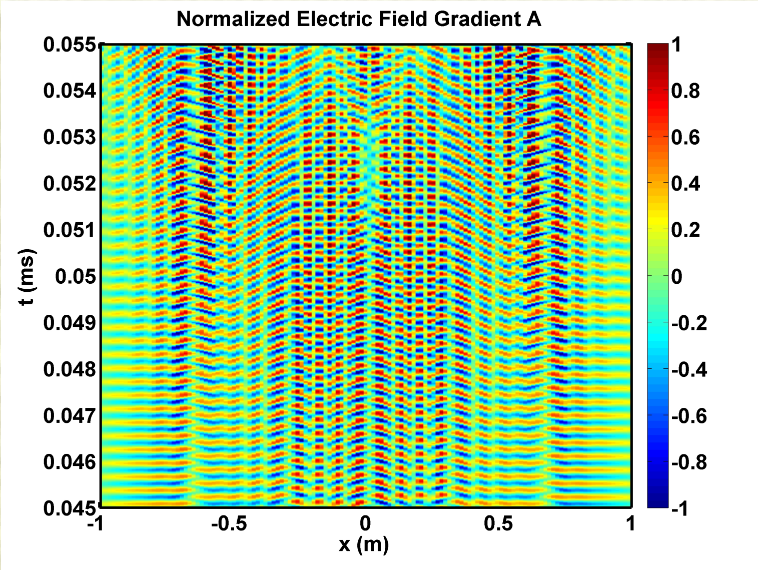
$E_0 = 2\,\mathrm{V/m}$. Hydrogen ions.

- Mode-converted upper hybrid (UH) waves (~ 50 cm) trapped in striation.

Mode conversion to UH waves, generation of EB waves

$$A = \frac{m_e}{eB_0^2}\frac{\partial E_x}{\partial x}$$ Normalized electric field gradient

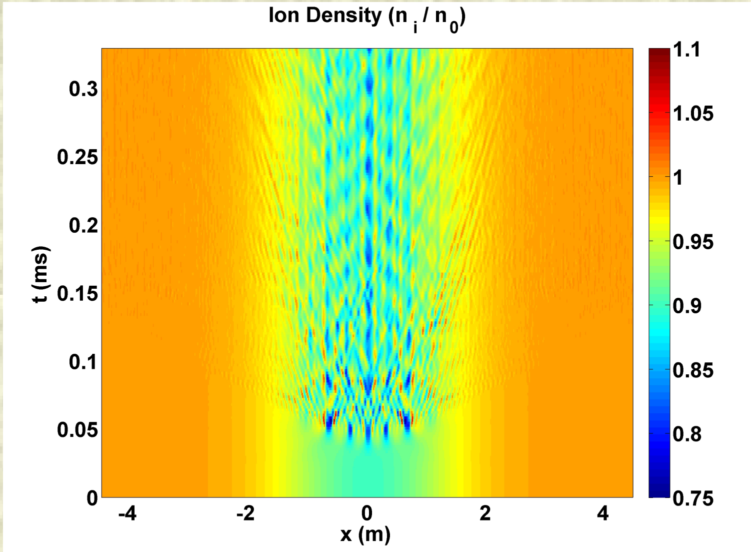


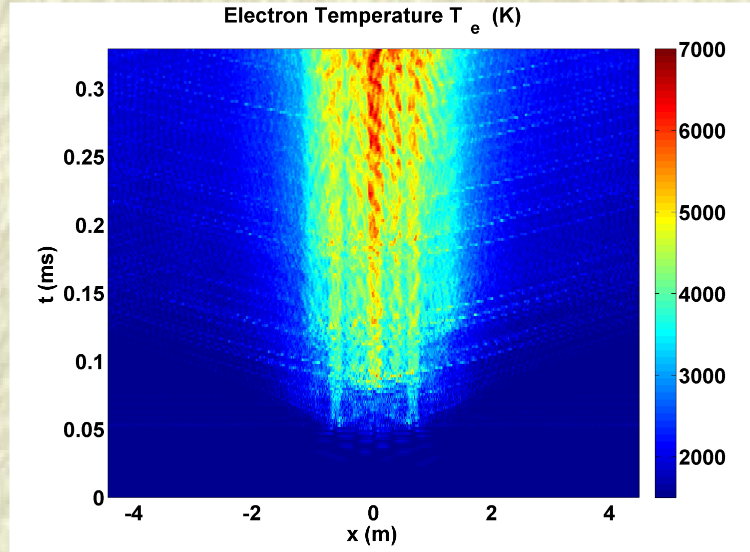


- Short wavelength electron Bernstein (EB) waves (~ 10 cm) excited and leaving the striations.
- Amplitude $|A| > 1$ exceeds threshold for stochastic heating.

A. Najmi, B. Eliasson et al., Radio Science (in press 2016).

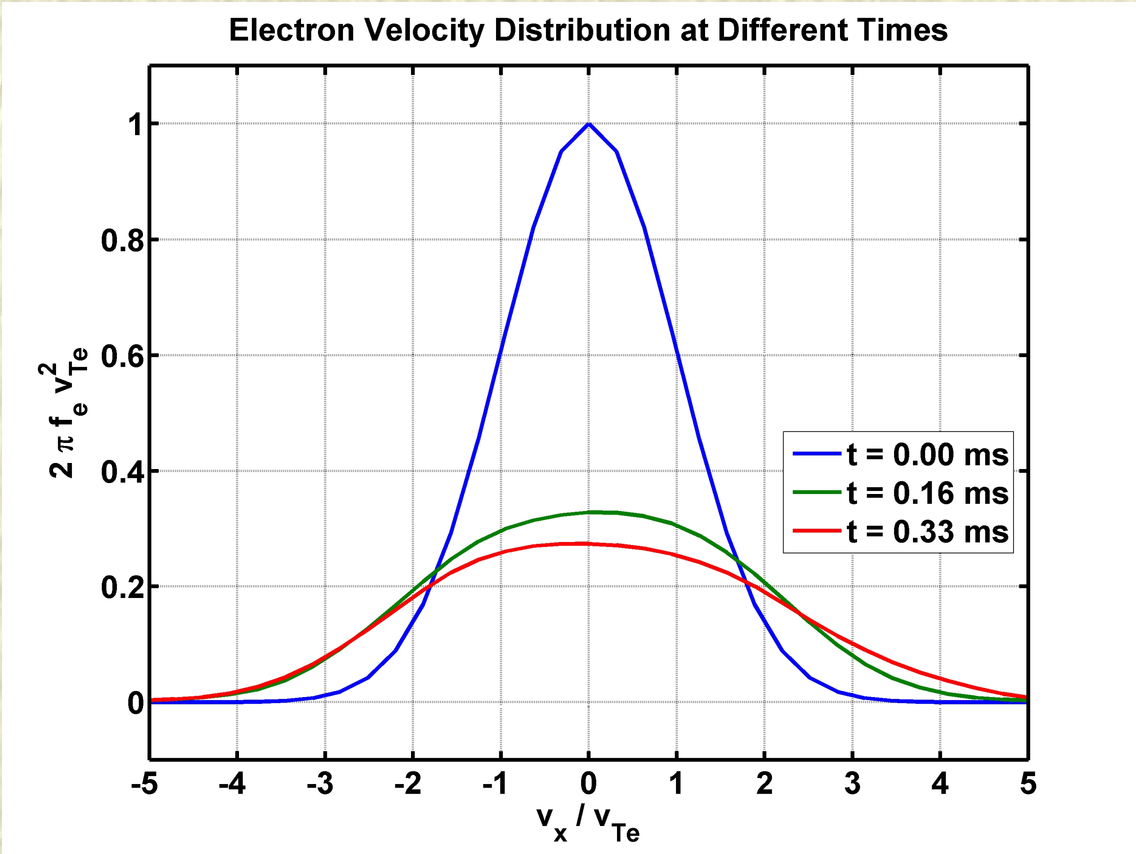
Lower hybrid oscillations and electron heating





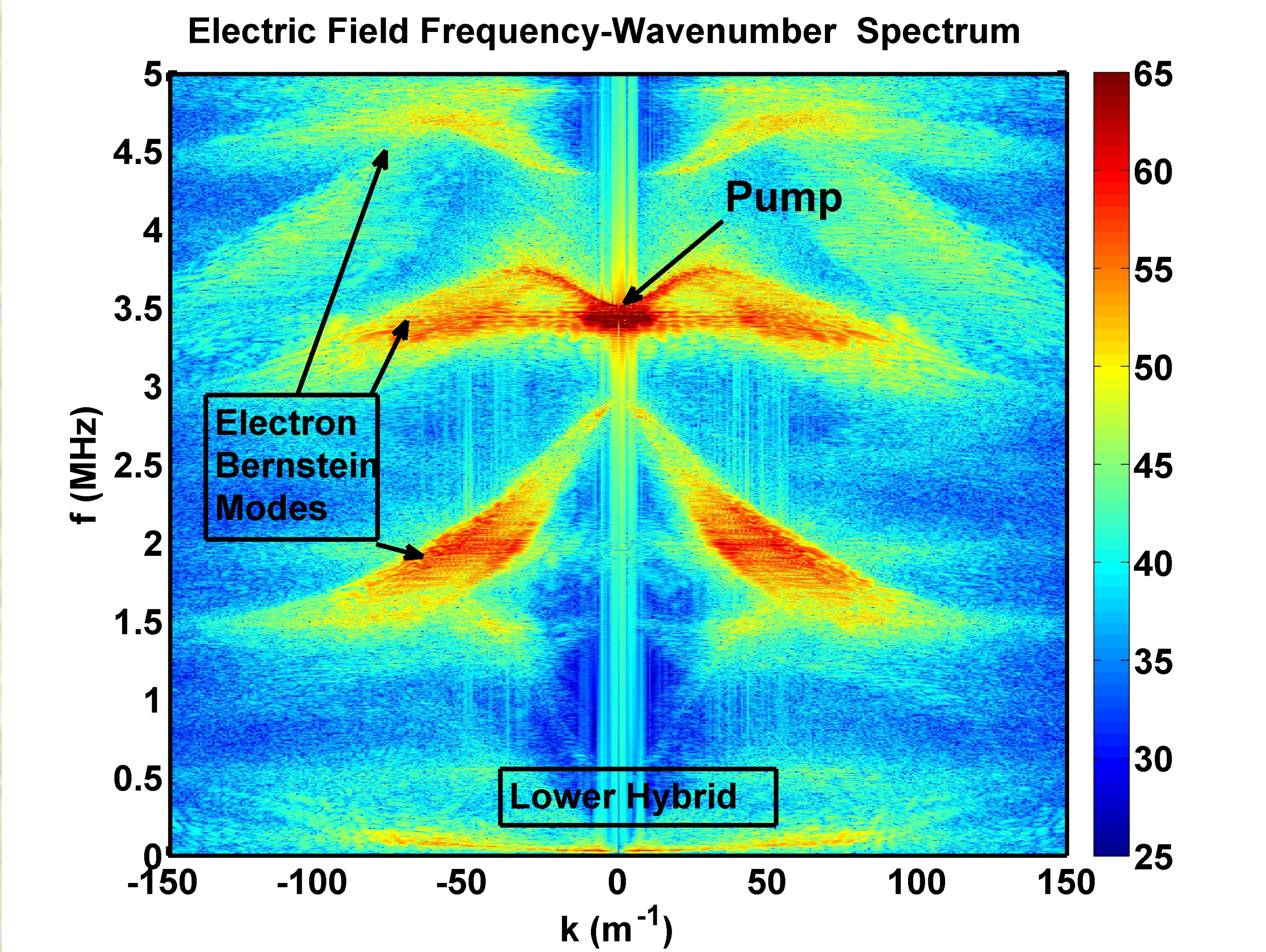
- Lower hybrid (LH) waves form standing wave pattern.
- Electron temperature rises to about 7000 K in the center of the striation.

Electron distribution function at different times



Electron distribution is flattened and widened — bulk heating but no high-energy tails.

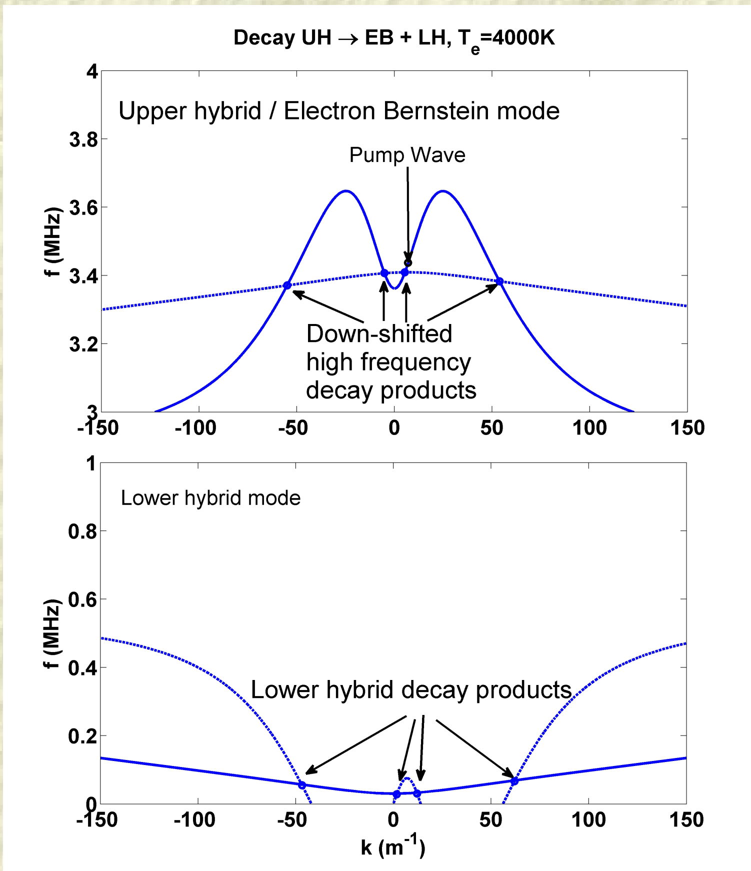
Coupling upper hybrid waves to EB and LH waves

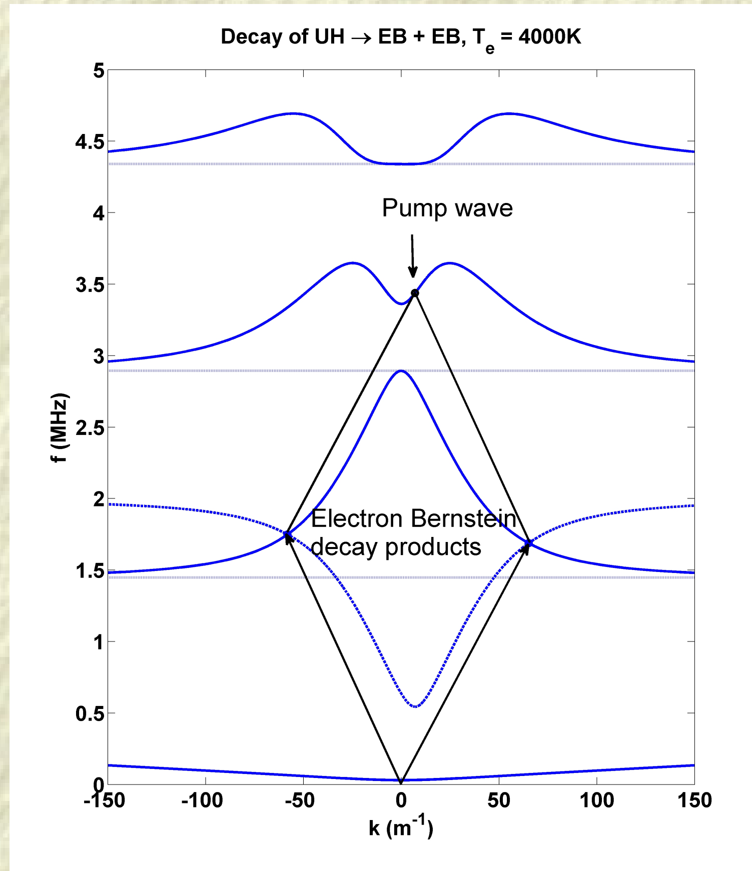


First three electron Bernstein modes and lower hybrid waves are visible.

3-wave decay scenarios

Matching conditions: $\omega_0 = \omega_1 + \omega_2, \quad \mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$





Also potentially 4-wave decay and UH wave collapse taking place.

Comparison: Stochastic heating by an electrostatic wave

Equations of motion for an electron in an electrostatic wave perpendicular to the magnetic field

$$m\frac{d\mathbf{v}^{(j)}}{dt} = -eE_0\sin(kx^{(j)} - \omega t)\widehat{\mathbf{x}} - e\mathbf{v}^{(j)} \times B_0\widehat{\mathbf{z}}, \qquad \frac{dx^{(j)}}{dt} = v_x^{(j)}$$

Normalized model equations

$$\frac{du_x^{(j)}}{dt} = -A\sin(u_y^{(j)} - \Omega t) - u_y^{(j)}, \qquad \frac{du_y^{(j)}}{dt} = u_x^{(j)}$$

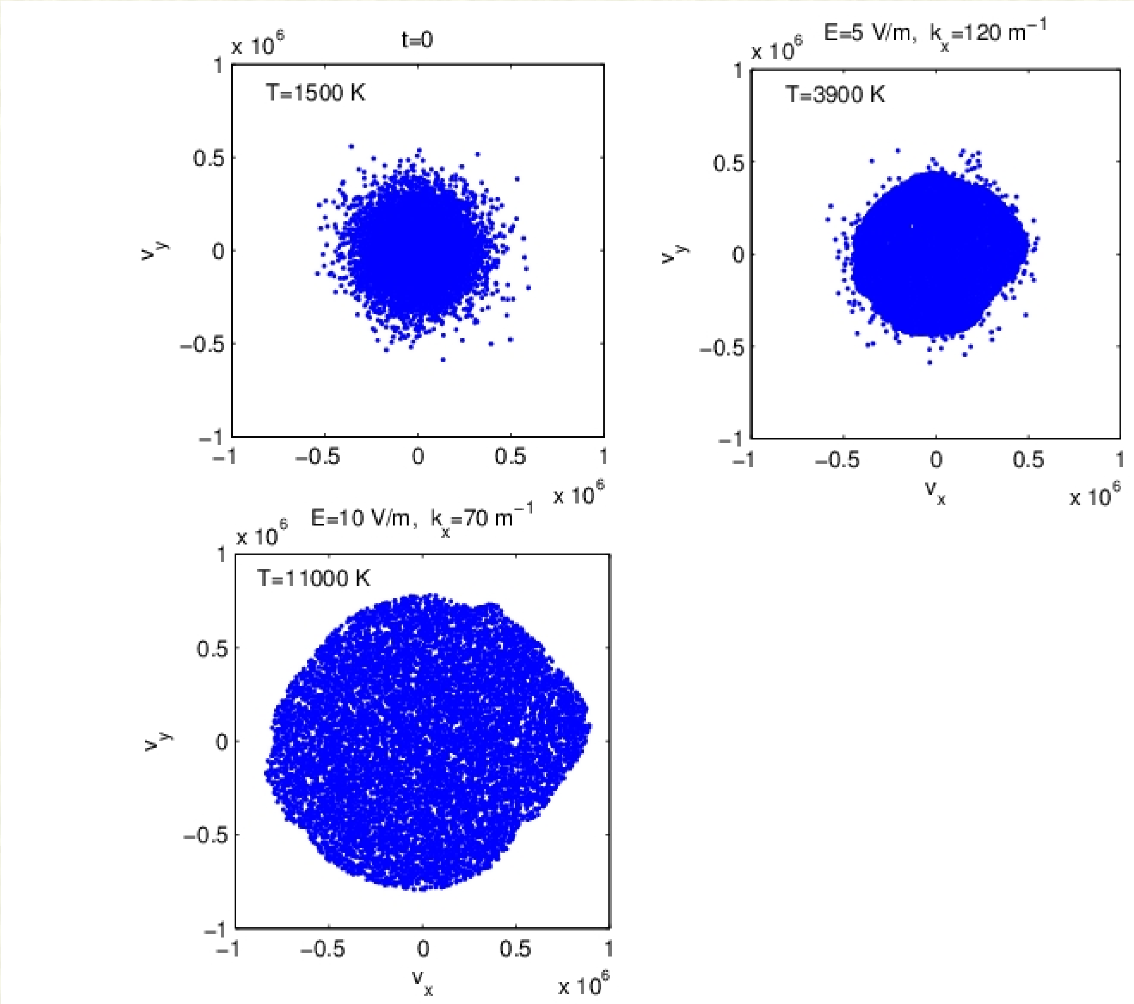
where $A = \frac{mkE_0}{eB_0^2}$ and $\Omega = \omega/\omega_{ce}$, $\omega_{ce} = eB_0/m$. Typically $A > 1$ leads to stochastic motion of the particles and to rapid heating of the plasma.

Has been extensively studied in the past:

M. Balikhin et al., Phys. Rev. Lett. **70**, 1259 (1993). $\rightarrow$ Electron heating by shocks
J. McChesney et al., Phys. Rev. Lett., **59**, 1436 (1987). $\rightarrow$ Ion heating by drift waves
C. F. F. Karney, Phys. Fluids **21**, 1584 (1978). $\rightarrow$ Ion heating by lower hybrid waves

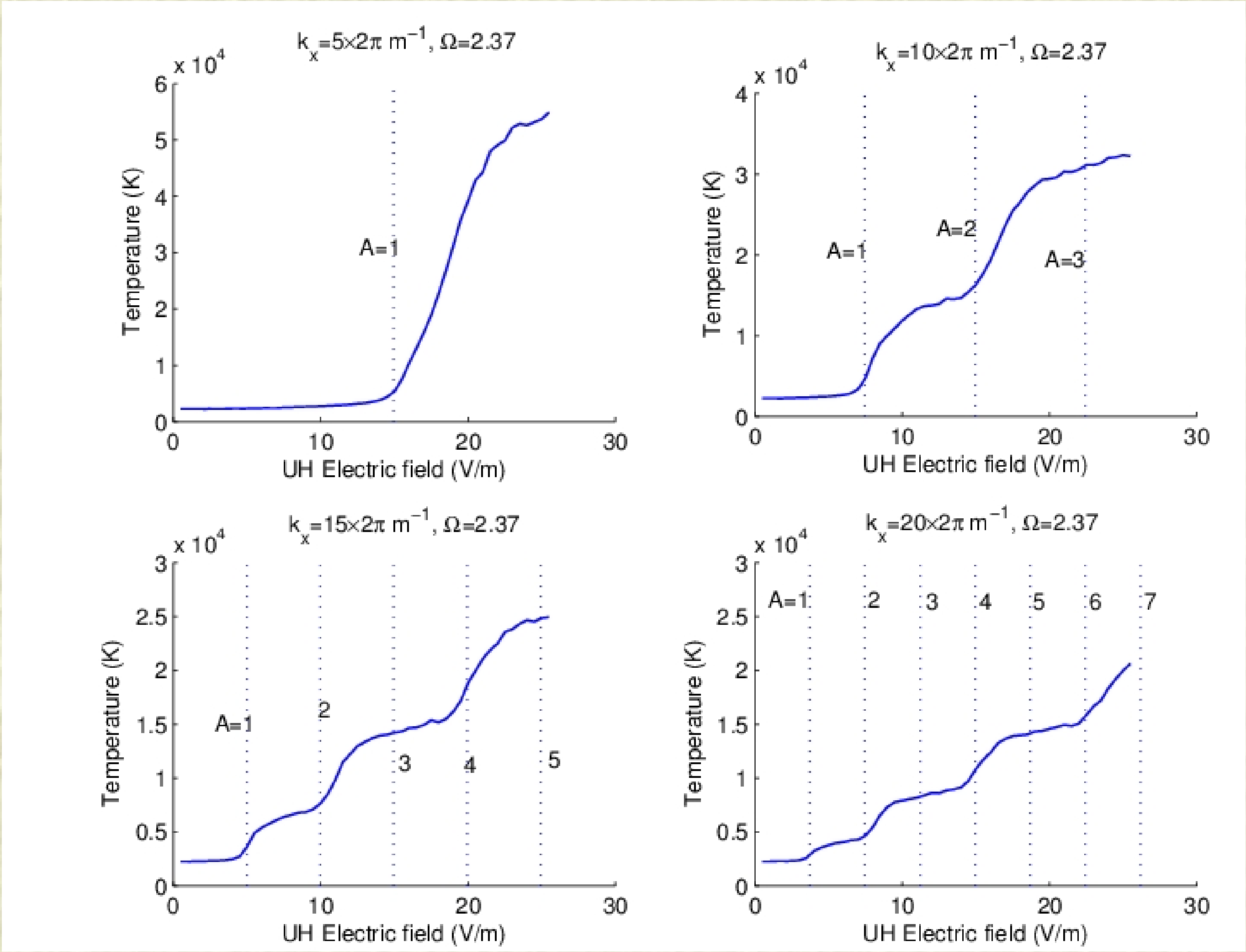
A. Fukuyama et al., Phys. Rev. Lett. **38**, 701 (1977) $\rightarrow$ Ion heating near gyroharmonics.

Electron distribution function



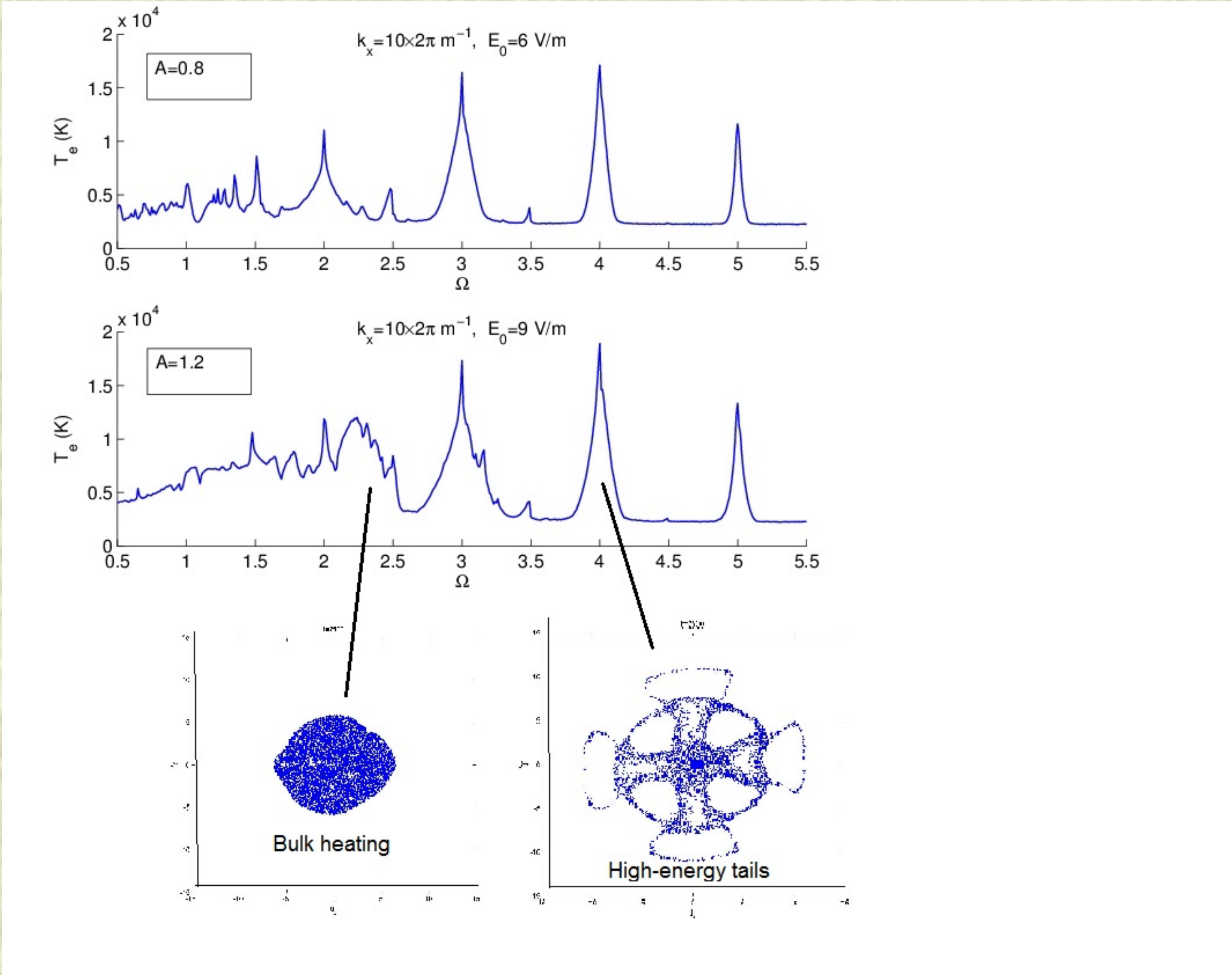
Test particle simulations 10^4 particles, simulation times a few hundred gyroperiods. Flat-topped electron distributions are developed. No suprathermal tails.

Temperature dependence on amplitude



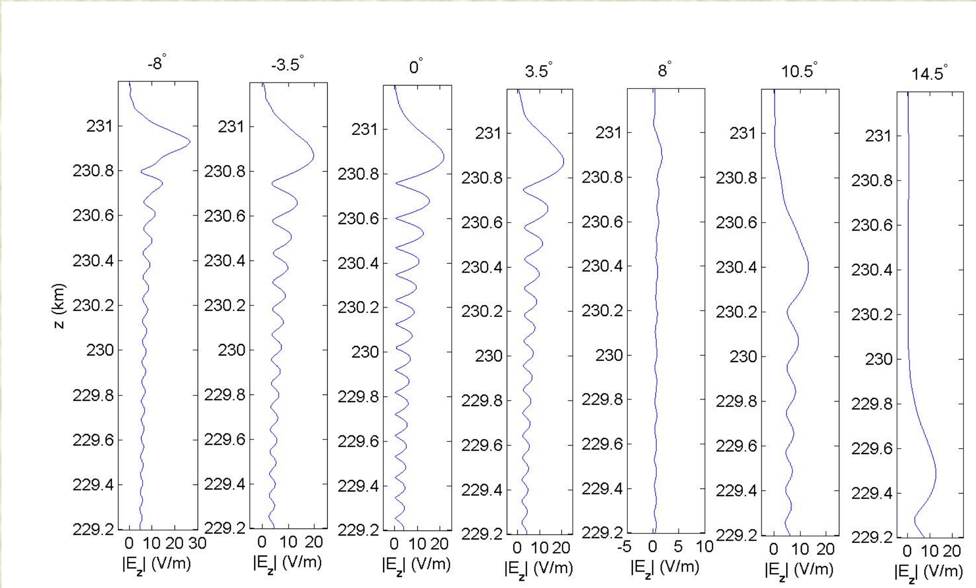
Each point on the curve represents one test particle simulation.

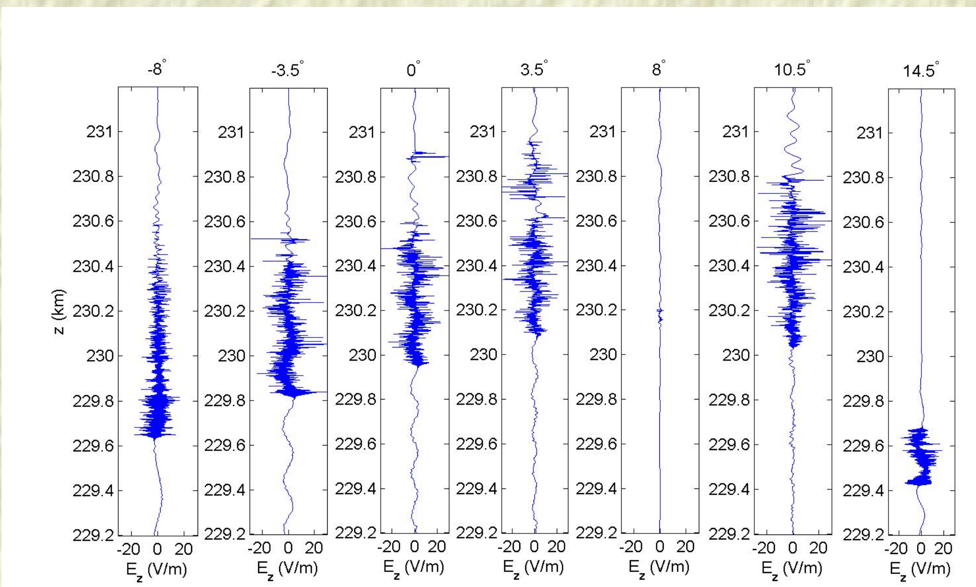
Temperature dependence on frequency



Temperature peaks near cyclotron harmonics. Rises between cyclotron harmonics for $A > 1$.

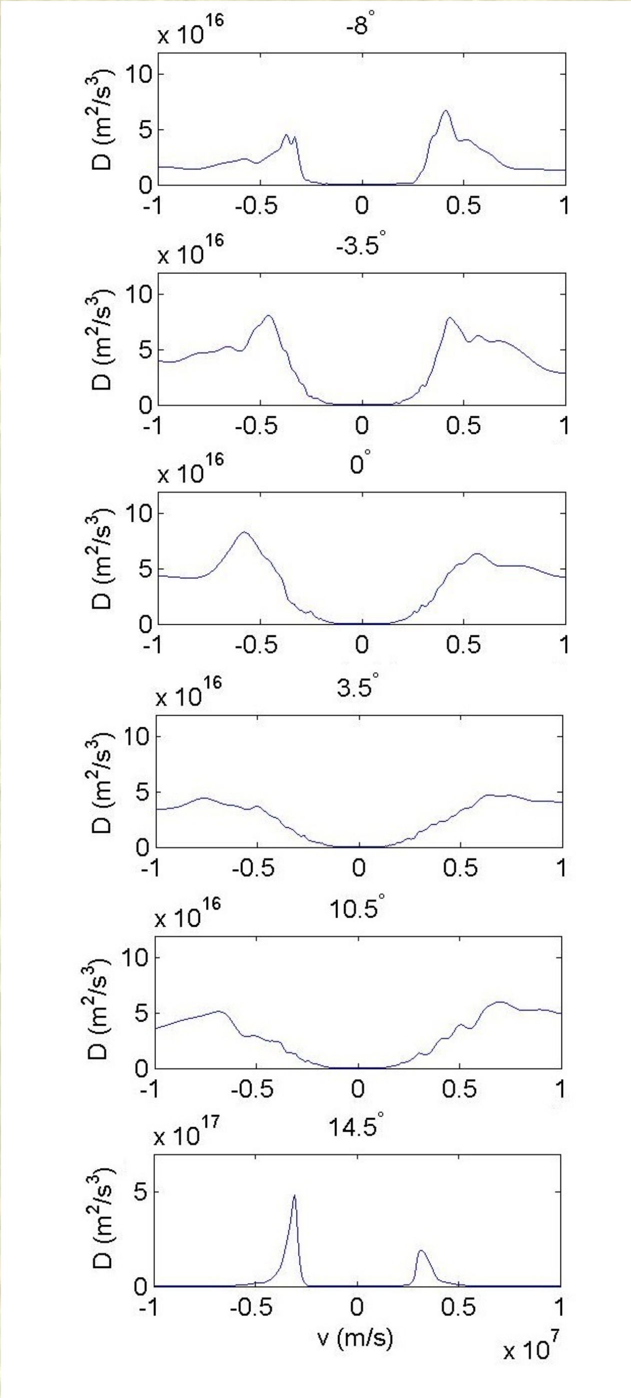
Electron acceleration by strong Langmuir turbulence

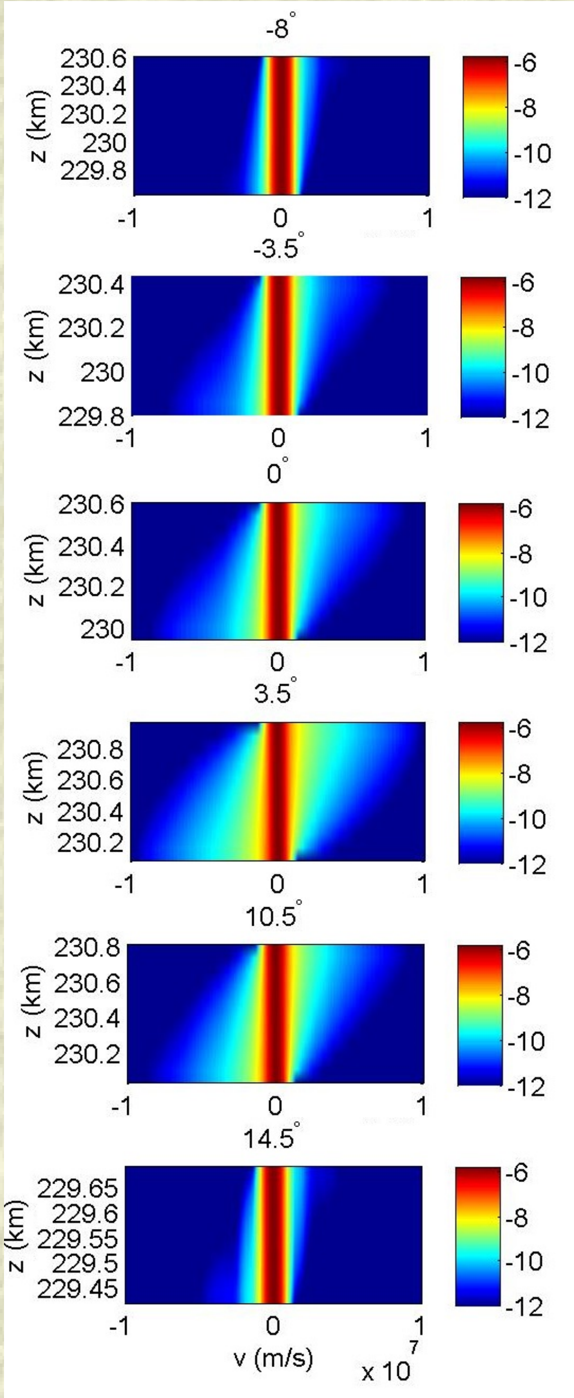




Electromagnetic wave breaks up into small-scale electromagnetic turbulence via parametric instabilities creating strong Langmuir turbulence

Most important:
4-wave oscillating two-stream instability creating localized wave envelopes accelerating electrons

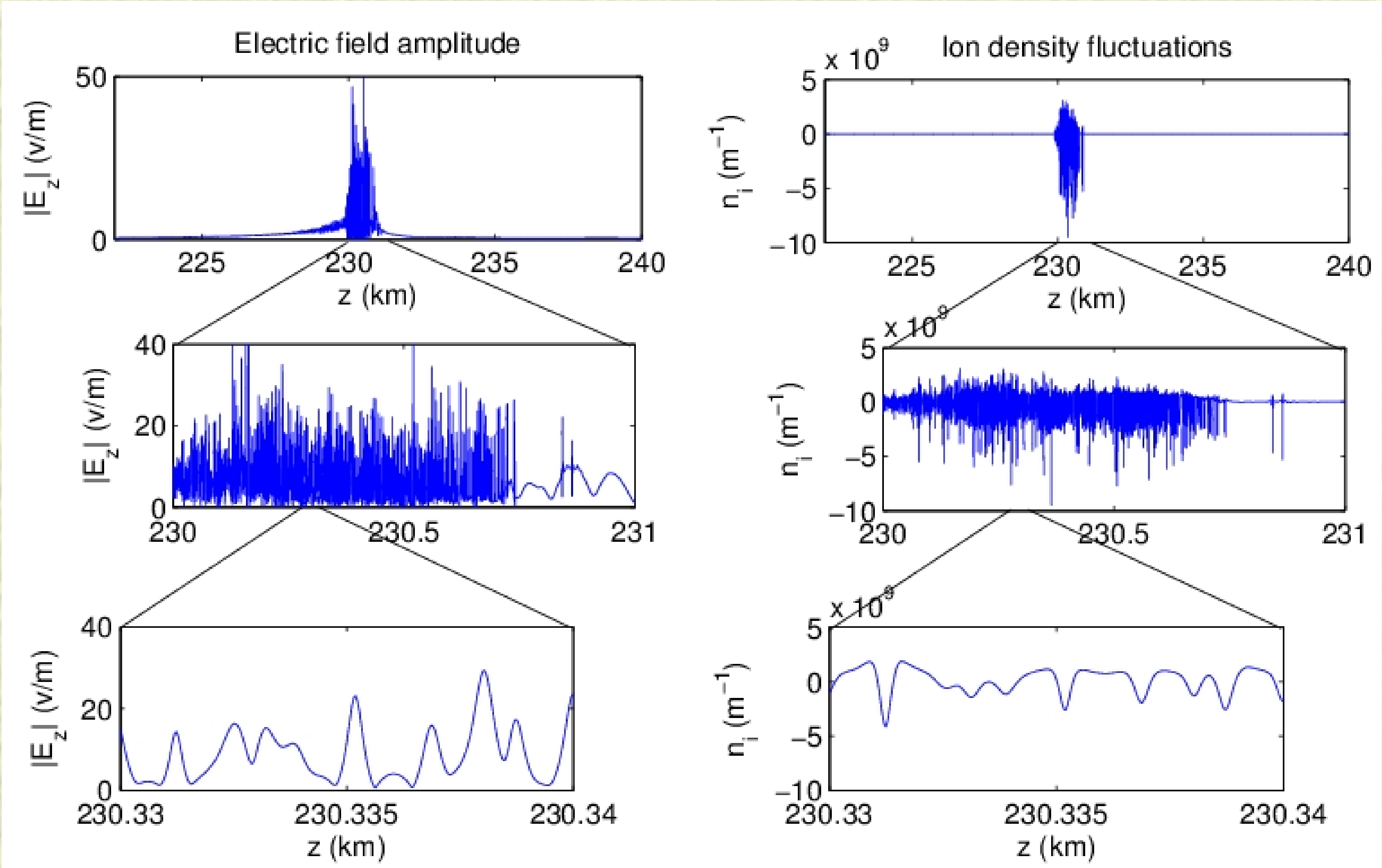




Diffusion coefficients and Fokker-Planck solutions (velocity distribution) for different angles of incidence

Most significant acceleration at $3.5°$ **and** $10.5°$

Physics at different length-scales



Small-scale strong Langmuir turbulence: few tens of centimetre structures.

Large amplitude electric field envelopes trapped in density cavities.

Some notes about the Vlasov simulations

Electron Vlasov-Poisson system with stationary ions

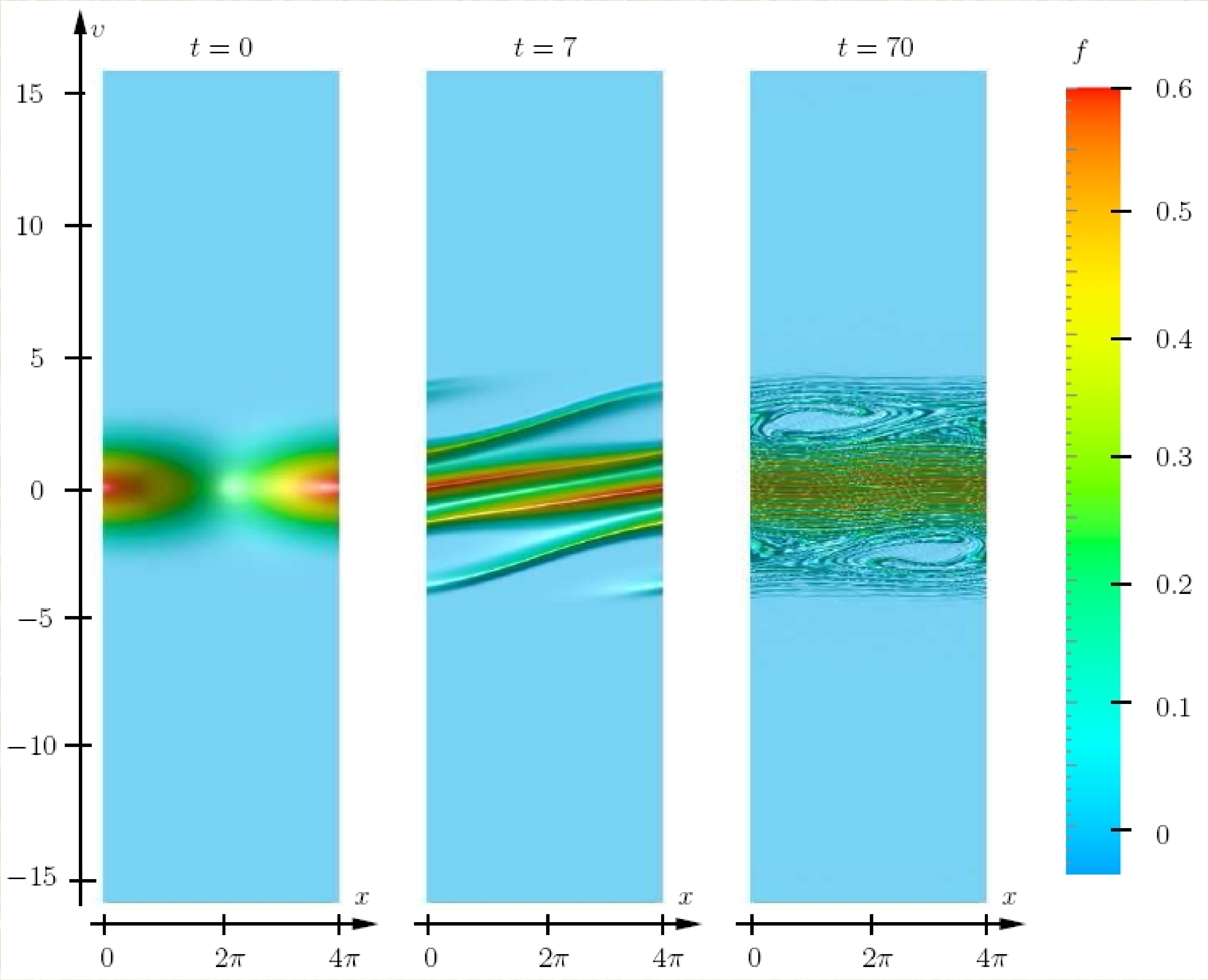
$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} - E\frac{\partial f}{\partial v} = 0$$
$$\frac{\partial E}{\partial x} = 1 - n_e$$
$$n_e = \int_{-\infty}^{\infty} f(x, v, t)\, dv$$

Initial condition

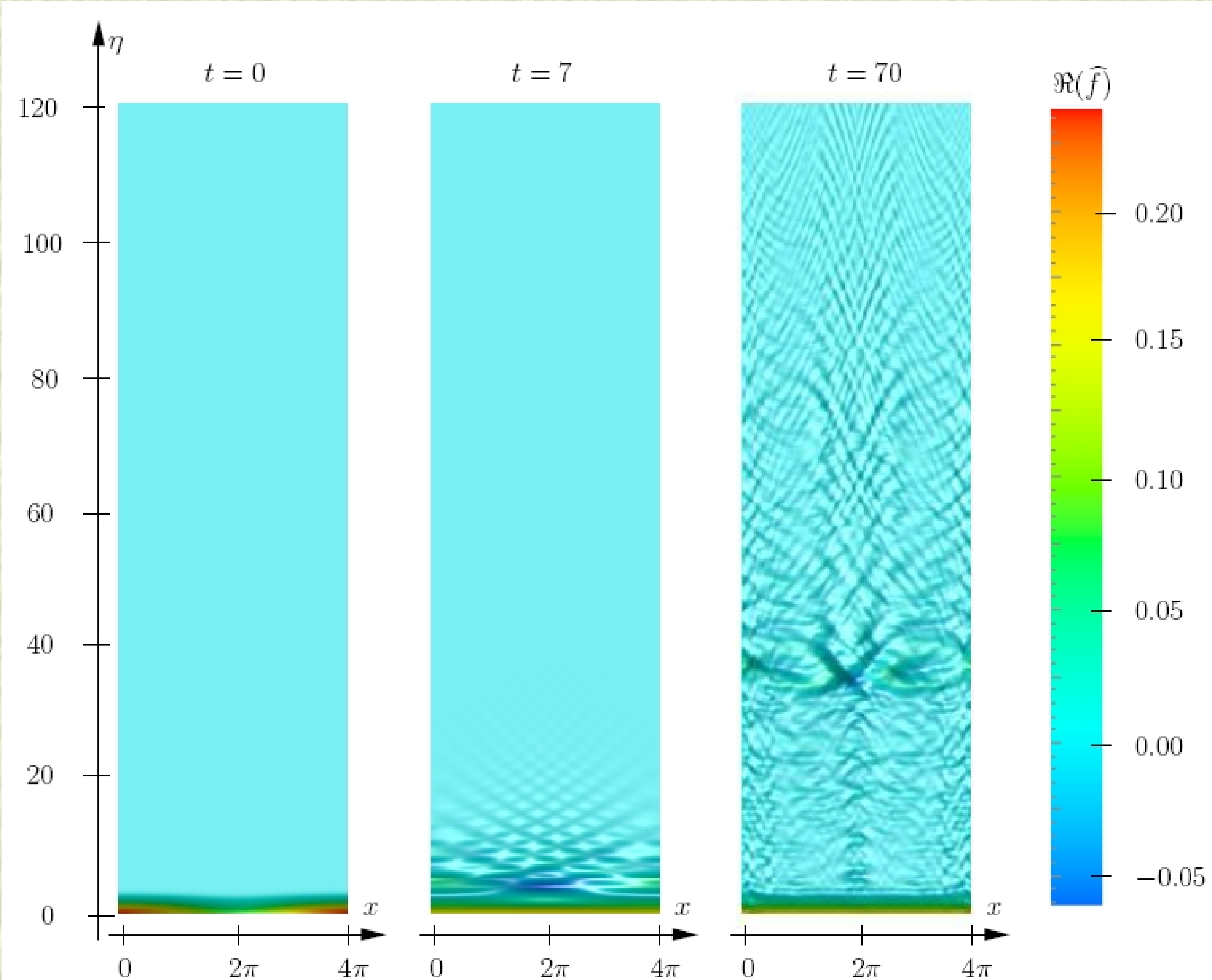
$$f(x, v, t = 0) = (2\pi)^{-1/2}[1 + A\cos(kx)]\exp(-v^2/2)$$

with $A = 0.5$, $k = 0.5$

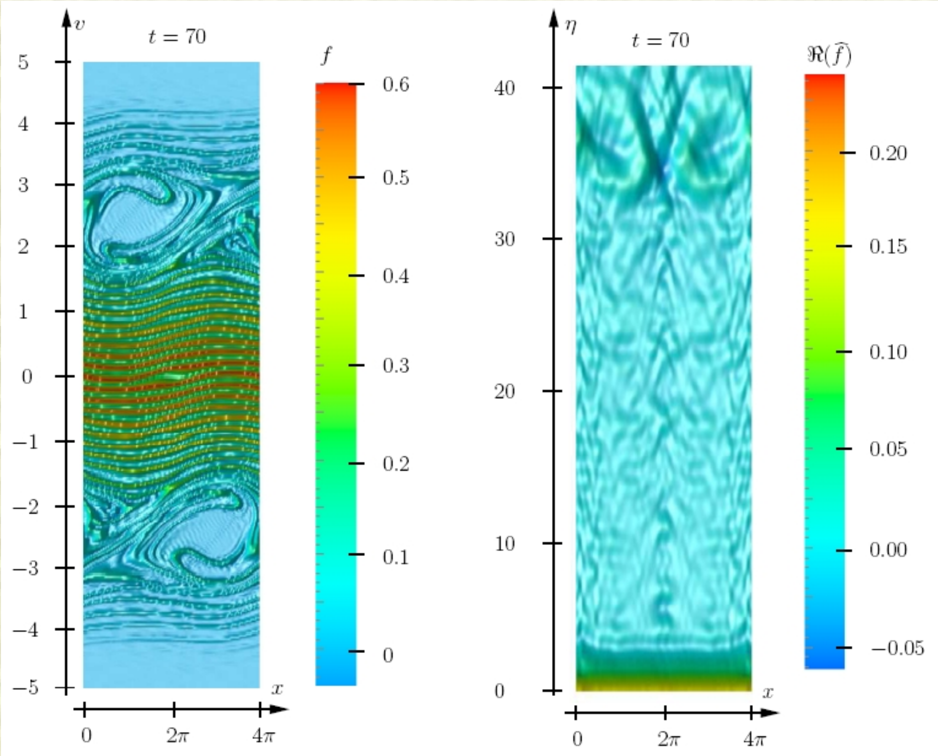
Electron phase space distribution



Fourier transformed velocity space



Closeup of solution



- Problems to calculate v derivatives and integrals numerically!
- Filamentation in v space gives rise to wave packet in η space.
- Strategy: Solve Vlasov equation Fourier transformed in velocity space.
- The highest harmonics in velocity space are allowed to propagate over the boundary at $\eta = \eta_{\max}$ and to be removed from the calculation.
- Introduces a minimum dissipation in velocity space: Very little numerical heating.

Fourier transformed Vlasov-Poisson system

(Stationary ions, normalized equations)

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} - E\frac{\partial f}{\partial v} = 0, \qquad \frac{\partial E}{\partial x} = 1 - \int_{-\infty}^{\infty} f(x,v,t)\,dv$$

The Fourier transform pair

$$f(x,v,t) = \int_{-\infty}^{\infty} \widetilde{f}(x,\eta,t)\mathrm{e}^{-i\eta v}\,d\eta, \qquad \widetilde{f}(x,\eta,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x,v,t)e^{i\eta v}\,dv$$

gives

$$\frac{\partial \widetilde{f}}{\partial t} - i\frac{\partial^2 \widetilde{f}}{\partial x \partial \eta} + E\eta\widetilde{f} = 0, \qquad \frac{\partial E(x,t)}{\partial x} = 1 - 2\pi\widetilde{f}(x,\eta,t)_{\eta=0}$$

Well-posed outflow boundary conditions for $\widetilde{f}$:

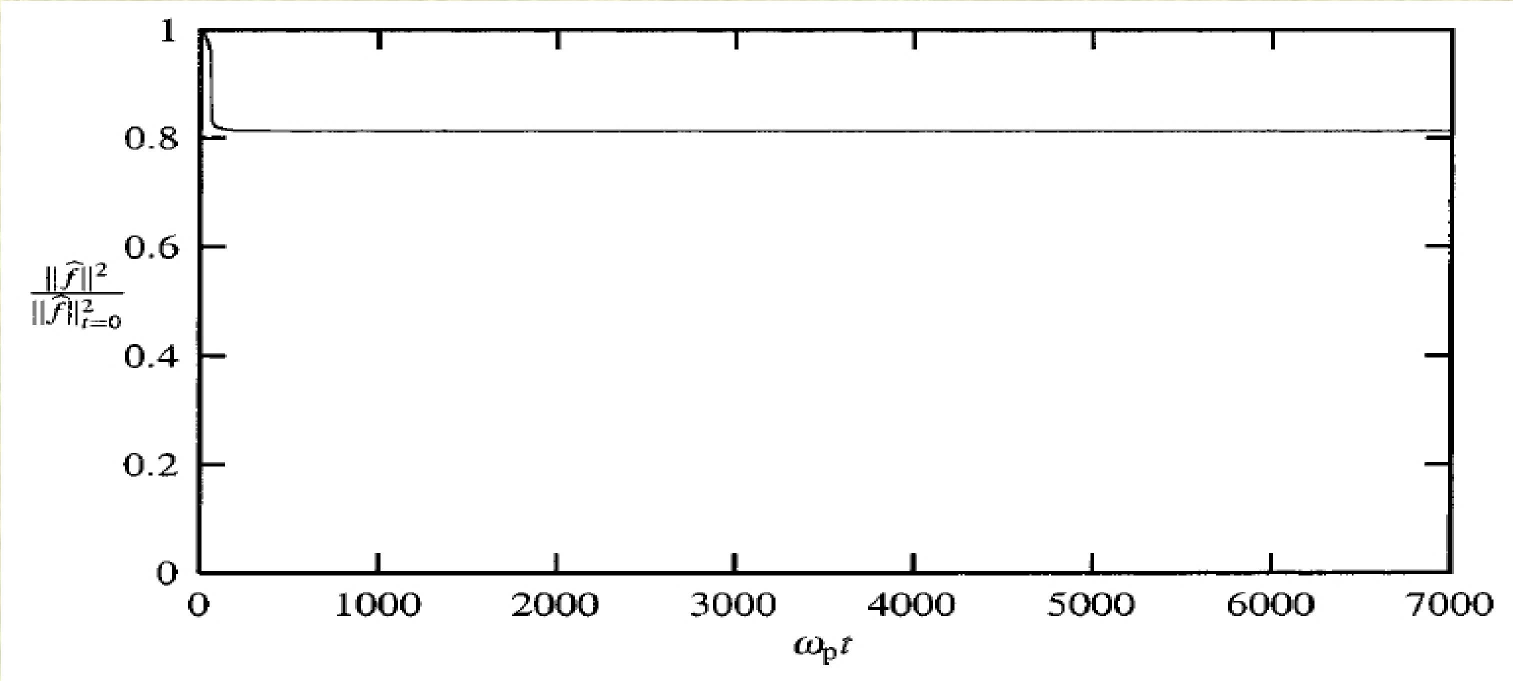
$$\widetilde{f} = \mathrm{F}^{-1}[H(k)\mathrm{F}\widetilde{f}] \quad \text{at } \eta = \eta_{max}$$

where F and F^{-1} are the forward and inverse spatial Fourier transforms.

What is flowing out at the outflow boundary?

With the outflow boundary conditions, one can show that the entropy-like functional is non-increasing

$$\frac{d}{dt}||\widetilde{f}||_2^2 = \frac{d}{dt}\int_0^L \int_{-\eta_{max}}^{\eta_{max}} |\widetilde{f}|^2 d\eta\, dx \leq 0$$



Holds also for the 2×2 and 3×3 dimensional Vlasov equations

Development of Vlasov code

❑ The Fourier method has been developed in 1×1, 2×2 and 3×3 dimensions.

☛ Electromagnetic and electrostatic options
☛ B. Eliasson, Transport Theory and Statistical Physics **39**, 387 (2011) [Proceedings of Vlasovia 2009]

❑ Fully parallelized in 1×1 and 2×2 dimensions (using MPI), working on parallelization in 3×3 dimensions.

☛ B. Eliasson, Comput. Phys. Commun. **170**, 205 (2005).
☛ L. K. S. Daldorff & B. Eliasson, Parallel Comput. **35**, 109 (2009).

❑ Various versions, including 3×3 hybrid-Vlasov, 2×2 Darwin, 2×2 Wigner solvers.

Summary

- ❏ Formation of descending aurora/ionization fronts in experiments. Ionosphere used as a plasma laboratory!
- ❏ Wave-wave interactions: Mode conversion and parametric instabilities creating short wavelength electrostatic waves
- ❏ Wave-particle interactions leading to acceleration of electrons
  - ☛ Stochastic heating. Large amplitude electron Bernstein waves perpendicular to the magnetic field makes the particle orbits unstable, leading to bulk heating of electrons
  - ☛ ”Quasilinear” acceleration: Diffusion in velocity space by strong Langmuir turbulence along magnetic field leading to the formation of high-energy tails.
- ❏ Vlasov simulations used to electron heating by Bernstein waves
- ❏ Physics on different length-scales tens of km to 0.1 m, and time-scales microseconds to minutes.