Forced hybrid-kinetic turbulence in 2D3V

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1. Motivation

2. The hybrid Vlasov-Maxwell (HVM) model

3. Results
Solar wind (SW) turbulence below the ion gyroradius

SW in-situ satellite measurements of turbulent energy spectra

- **large scales**: magnetohydrodynamic (MHD) inertial range $\rightarrow \sim k_{\perp}^{-5/3}$ spectrum.
- **first spectral break** at ions’ characteristic scales ($k_{\perp} \rho_i \sim 1$ and/or $k_{\perp} d_i \sim 1$).
- “dissipation/dispersion” range ($1 \lesssim k_{\perp} \rho_i \lesssim \rho_i / \rho_e$):
  - $\rightarrow$ **B-field spectrum**: slope in the range $[-2.5, -3]$.
  - $\rightarrow$ **E-field spectrum**: slope in the range $[-0.3, -1.3]$ ($\rightarrow$ noise?).
  - $\rightarrow$ energy in the E-field overcomes the magnetic counterpart.
Solar wind (SW) turbulence below the ion gyroradius

**SW in-situ satellite measurements of turbulent energy spectra**

A long-lasting debate and open problem in SW turbulence research:

what is the nature of turbulent fluctuations below ion kinetic scales?

[Sahraoui et al., PRL 102 (2009)]

[Alexandrova et al., Space Sci Rev 178 (2013)]
Solar wind (SW) turbulence below the ion gyroradius

Theoretical candidates:

**kinetic Alfvén waves (KAWs)**


\[
E_B(k_\perp) \propto k_\perp^{-7/3}
\]

\[
E_E(k_\perp) \propto k_\perp^{-1/3}
\]

**whistler waves**

[Galtier & Bhattacharjee, PoP 10, 3065 (2003)]

\[
E_B(k_\perp) \propto k_\perp^{-7/3}
\]

\[
E_E(k_\perp) \propto k_\perp^{-1/3}
\]

Same spectra, but different physics

\[\downarrow\]

auxiliary methods to distinguish between them

Possible sources of steepening:

- Landau damping  
  [Howes et al., JGR 113 (2008)]

- Compressibility effects:  
  \[E_B \propto k_\perp^{-7/3 - 2\xi}\]

- Intermittency corrections:  
  \[E_B \propto k_\perp^{-8/3} \text{ and } E_E \propto k_\perp^{-2/3}\]
Solar wind (SW) turbulence below the ion gyroradius

Numerical simulations: reproducing energy spectra

3D GK driven KAWs [Howes et al., PRL 107 (2011)]


3D HVM freely-decaying [Servidio et al., JPP 81 (2015)]

So far, freely-decaying simulations and/or focus on one scenario at a time
Hybrid Vlasov-Maxwell (HVM) model

Fully kinetic ions & massless electron fluid:

\[
\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \frac{\partial f_i}{\partial \mathbf{x}} + (\mathbf{E} + \mathbf{v} \times \mathbf{B} + \mathbf{F}) \cdot \frac{\partial f_i}{\partial \mathbf{v}} = 0 \quad \text{(Vlasov equation)}
\]

\[
\mathbf{E} = -\mathbf{u}_i \times \mathbf{B} + \frac{1}{n} (\mathbf{J} \times \mathbf{B}) - \frac{1}{n} \nabla P_e + \eta \mathbf{J} + \mathcal{O} \left( \frac{m_e}{m_i} \right) \quad \text{(gener. Ohm’s law)}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \nabla \times \mathbf{B} = \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \quad \text{(Maxwell’s equations)}
\]

\[
\mathbf{F} = \mathbf{F}(\mathbf{x}, t): \text{ random forcing, } \delta\text{-correlated in time.}
\]

\[
m_e = 0, \quad n_i = n_e = n, \quad \omega/k \ll c, \quad P_e = nT_{e0}
\]
Simulations setup

- **2D-3V phase space:**

  \[1024^2 \times 51^3\] grid points \((k_\perp d_i \in [0.1, 51.2])\)

- **Initial condition:**

  \[f_i(x, v, t = 0) = \text{isotropic Maxwellian}\]

  \[B(x; t = 0) = B_0 e_z + \delta B(x) \quad (|\delta B| \ll B_0 \text{ and } 0.1 \leq (k_\perp d_i)_{\delta B} \leq 0.3)\]

- **F injection scale:**

  \[0.1 \leq (k_\perp d_i)_F \leq 0.2\] (continuously forced)

  \[\rightarrow \text{forcing contributions: } \sim 50\% \text{ compressible, } \sim 50\% \text{ incompressible}\]

- **Beta regimes investigated:**

  \[\beta = 0.2, 1 \text{ and } 5\]
The quest for a compromise: model & setup

**Major “weak” points**
- reduced dimensionality (2D) of the simulations
- electron Landau damping (LD) is missing on all modes

**Major “strong” points**
- in 2D we can include three decades in the spectra
- fully kinetic ions (e.g., ion cyclotron resonances are included)
- we do not focus on a particular mode (both KAWs and whistler are allowed)
- $F$ allows to reach a quasi-steady turbulent state
- the growth of in-plane magnetic fluctuations allows for $k_{||} \neq 0$

*we expect these “2.5D” simulations to retain some important dynamical features of the fully 3D case*
Developing plasma turbulence ($J_z$)

Example of $J_z$ contours for $\beta_i = 1$, at $\Omega_{ci}t = 120$ (left) and $\Omega_{ci}t = 225$ (right).

- formation of small-scale structures $\rightarrow$ kinetic regime
- current sheets $\rightarrow$ magnetic reconnection $\rightarrow$ fully developed turbulence
Developing plasma turbulence ($B_\perp$)

Example of $B_\perp$ contours and $A_z$ lines for $\beta_i = 1$, at $\Omega_{ci}t = 120$ (left) and $\Omega_{ci}t = 225$ (right).

- in-plane magnetic fluctuations: randomly oriented, $\langle B_\perp \rangle < 0.1$
- local high-$B_\perp$ spots: current sheets, coherent structures
Developed plasma turbulence ($E_\perp$)

Contours of $E_{\text{MHD}}$ (left) and of $E_{\text{Hall}}$ (right) for $\beta_i = 1$ at $\Omega_{ci} t = 225$.

- $E_{\text{MHD}} = u_i \times B$ dominates at large-scales (left)
- $E_{\text{Hall}} = (J \times B)/n$ dominates at small-scales, inside current sheets (right)
Magnetic energy spectrum

- $k_{\perp} d_i < 1$: Kolmogorov-type $k_{\perp}^{-5/3}$ spectrum
- spectral break at $1 \lesssim k_{\perp} d_i \lesssim 2$
- $k_{\perp} d_i > 1$: consistent with $k_{\perp}^{-8/3}$ at $\beta = 0.2, 1$ ($k_{\perp}^{-3}$ at $\beta_i = 5$)
Electric energy spectrum

- Electric energy overcomes magnetic counterpart at $k_{\perp}d_i \sim 2$

- Spectral slopes generally steeper than theory predictions
  (observed in other simulations and some SW measurements $\rightarrow$ feedbacks?)
KAWs or whistlers? (Auxiliary method I)

**Auxiliary method I:**

[Chen et al., PRL 110, 225002 (2013)]

comparing the level of $E_B$ and $C_0 E_n$

(with $C_0 = [\beta_i(1 + \tau)/2][1 + \beta_i(1 + \tau)/2]$)

- **KAWs** → $C_0 E_n \simeq E_B$.
- **whistlers** → $C_0 E_n \ll E_B$.

![Graphs showing the comparison between $E_B$ and $C_0 E_n$ for different values of $\beta$.](image)
**Auxiliary method II:**


**KAWs** fluctuations would obey the following relation:

\[ C_1 E_n = E_B || \]

(with \( C_1 = [\beta_i (1 + \tau) / 2]^2 \))
Partially compressible vs incompressible injection ($\beta = 0.2$)

Partially compressible forcing ($\nabla \cdot \mathbf{F} \neq 0$):

$$\langle E_{B\parallel}(k_{\perp}) \rangle, C_1 \langle E_n(k_{\perp}) \rangle$$

$\beta = 0.2$

$\rightarrow$ well separated even at $k_{\perp} \rho_i > 1$

Purely incompressible forcing ($\nabla \cdot \mathbf{F} = 0$):

$$\langle E_{B\parallel}(k_{\perp}) \rangle, C_1 \langle E_n(k_{\perp}) \rangle$$

$\rightarrow$ transition to KAWs at $k_{\perp} \rho_i \sim 1$
Conclusions

- **general agreement of spectral properties** of the turbulence (e.g., power laws and spectral breaks) with observations/theory.

- In this setup turbulence mainly involves **whistler fluctuations at low** $\beta$, and **KAWs at somewhat higher** $\beta$.

- **KAW $\leftrightarrow$ whistler turbulence transition**: possible correlation with resonant/non-resonant damping of the modes.
  (not straightforward: linear damping and/vs non-linear effects)

- **compressibility level** of injected fluctuations matters $\rightarrow$
  non-universality and possible implications on time and space variability of SW.

  $\rightarrow$ **call for further investigations on these topics...**
Thanks for your attention!