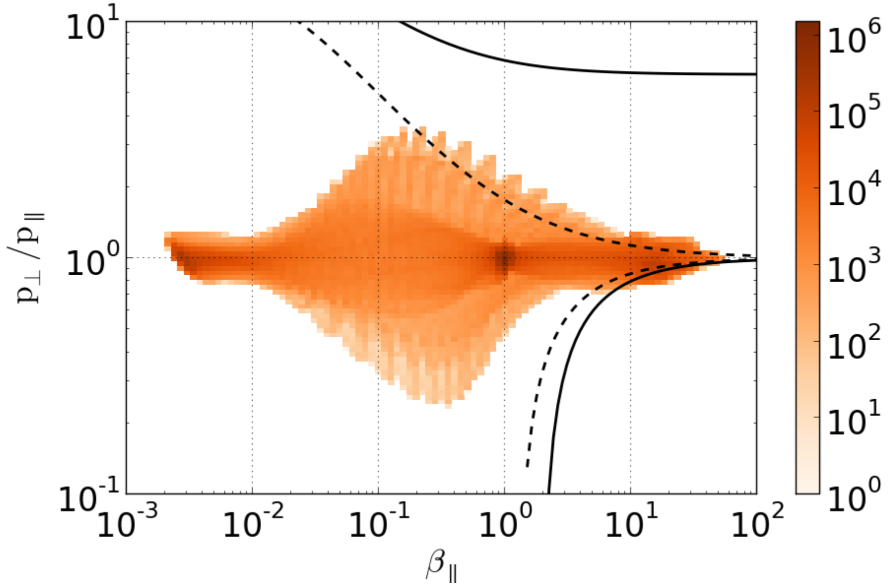
Physical mechanisms of regulation of pressure anisotropy in collisionless turbulent plasmas within MHD-CGL regime

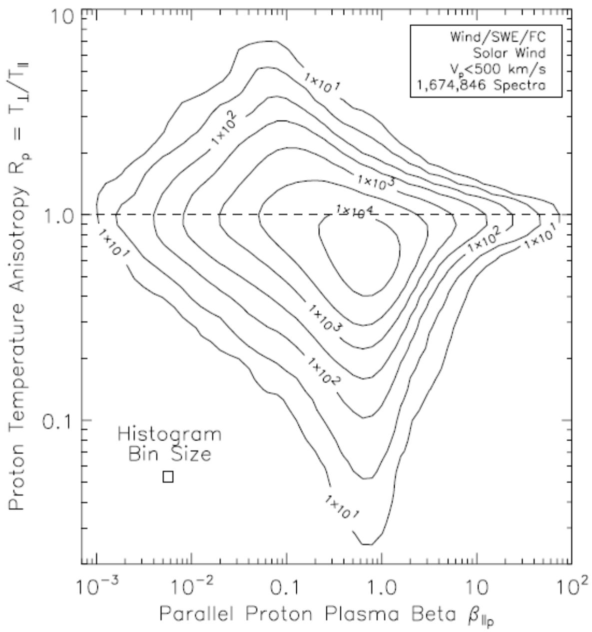


Marek Strumik[1], Alexander Schekochihin[1,2], Per Helander[3,1,2]

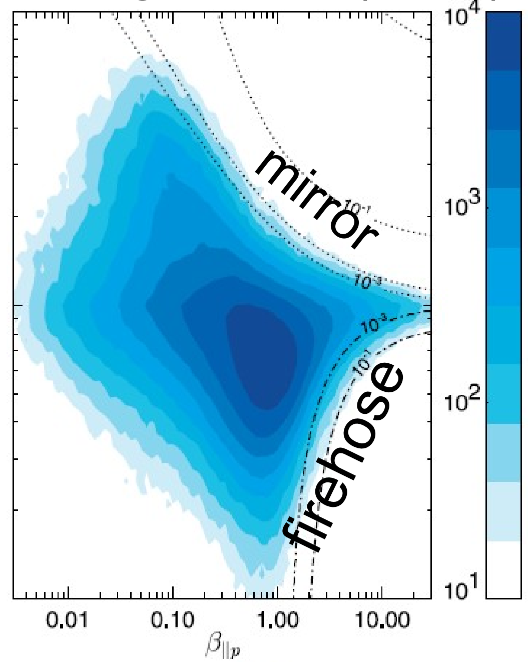
(1) Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford OX1 3NP, UK
(2) Merton College, Oxford OX1 4JD, UK
(3) Max-Planck-Institut fur Plasmaphysik, Wendelsteinstr. 1, 17491 Greifswald, Germany

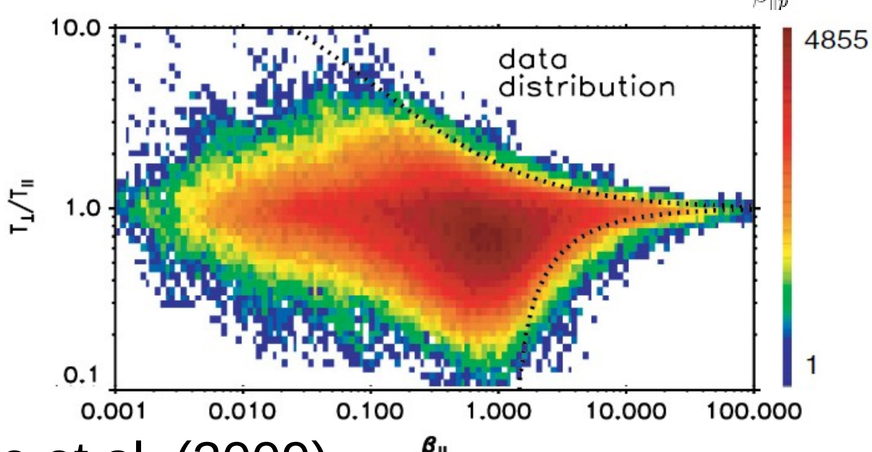
Pressure/temperature anisotropy distribution in slow solar wind

Kasper (2002)



Hellinger et al. (2006)





Bale et al. (2009)

Distribution of pressure anisotropy in turbulent slow solar wind in ($\beta_\parallel$,$T_\perp/T_\parallel$) plane.

Commonly believed to be constrained by the mirror instability for $T_\perp/T_\parallel$>1 and the firehose instability for $T_\perp/T_\parallel$<1.

Chew-Goldberger-Low (CGL, double-adiabatic) closure

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\perp}{\rho B}\right) = 0 \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\parallel B^2}{\rho^3}\right) = 0$$

This is equivalent to saying that

$$p_\perp \propto \rho B, \quad p_\parallel \propto \rho^3/B^2$$

in plasma frame (along streamlines)

Derivation of the CGL equations is also possible using two adiabatic invariants for particle motion

Chew-Goldberger-Low (CGL, double-adiabatic) closure

Evolutions of pressure components

$$\frac{\mathrm{d}\log p_\perp}{\mathrm{d}t} - (\hat{\mathbf{B}}\hat{\mathbf{B}}\cdot\nabla)\cdot\mathbf{u} + 2\nabla\cdot\mathbf{u} = -\nabla\cdot(q_\perp\hat{\mathbf{B}}) - q_\perp\nabla\cdot\hat{\mathbf{B}} = 0$$

$$\frac{\mathrm{d}\log p_\parallel}{\mathrm{d}t} + 2(\hat{\mathbf{B}}\hat{\mathbf{B}}\cdot\nabla)\cdot\mathbf{u} + \nabla\cdot\mathbf{u} = -\nabla\cdot[(q_\parallel + q_\perp)\hat{\mathbf{B}}] - 2\hat{\mathbf{B}}\cdot\nabla q_\parallel = 0$$

and density and magnetic field strength

$$\frac{\mathrm{d}\log B}{\mathrm{d}t} = (\hat{\mathbf{B}}\hat{\mathbf{B}}\cdot\nabla)\cdot\mathbf{u} - \nabla\cdot\mathbf{u}$$

$$\frac{\mathrm{d}\log\rho}{\mathrm{d}t} = -\nabla\cdot\mathbf{u}$$

are determined by velocity fluctuations $(\hat{\mathbf{B}}\hat{\mathbf{B}}\cdot\nabla)\cdot\mathbf{u}, \nabla\cdot\mathbf{u}$

Simulation setups

- **3D** simulations (resolutions 64^3-256^3, periodic boundary conditions), MHD equations with the **CGL closure**
- **Isotropic** pressure in the **initial condition**
- Stirring of plasma by large-scale random-phase **fluctuations of the magnetic field or velocity** (isotropic in wave vector space)
- Presence of **mean field**, **subsonic** regime

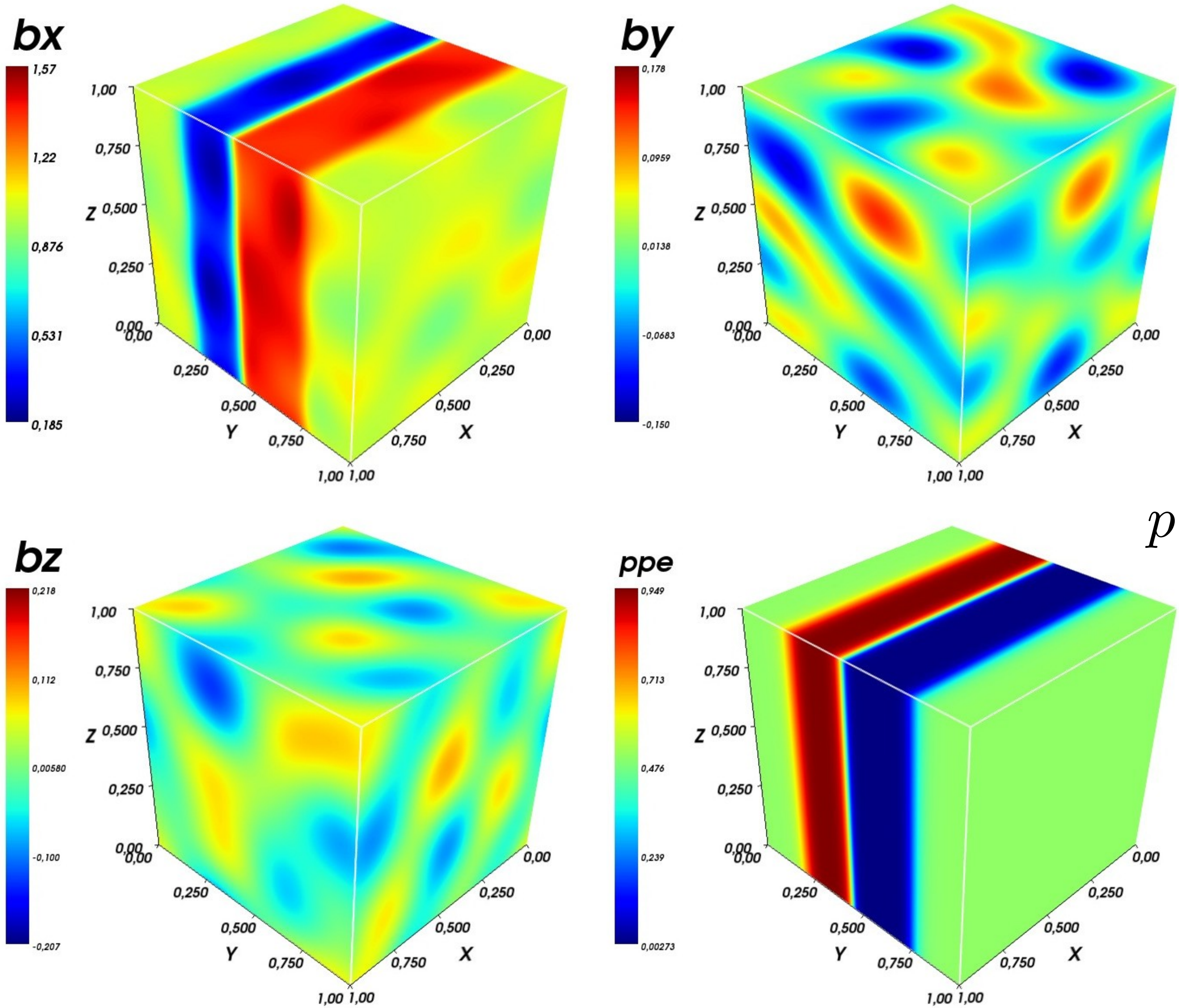
Magnetic relaxation

- initially large-scale magnetic field fluctuations
- large-scale pressure-balance structure to maintain wide range of β
- freely decaying turbulence

Forced randomized velocity fluctuations

- external random forcing by injection of large-scale velocity fluctuations
- β-scaling “scans” by changing average pressure

Magnetic relaxation, initial condition:
randomized fluctuations + pressure-balance structure



Initially

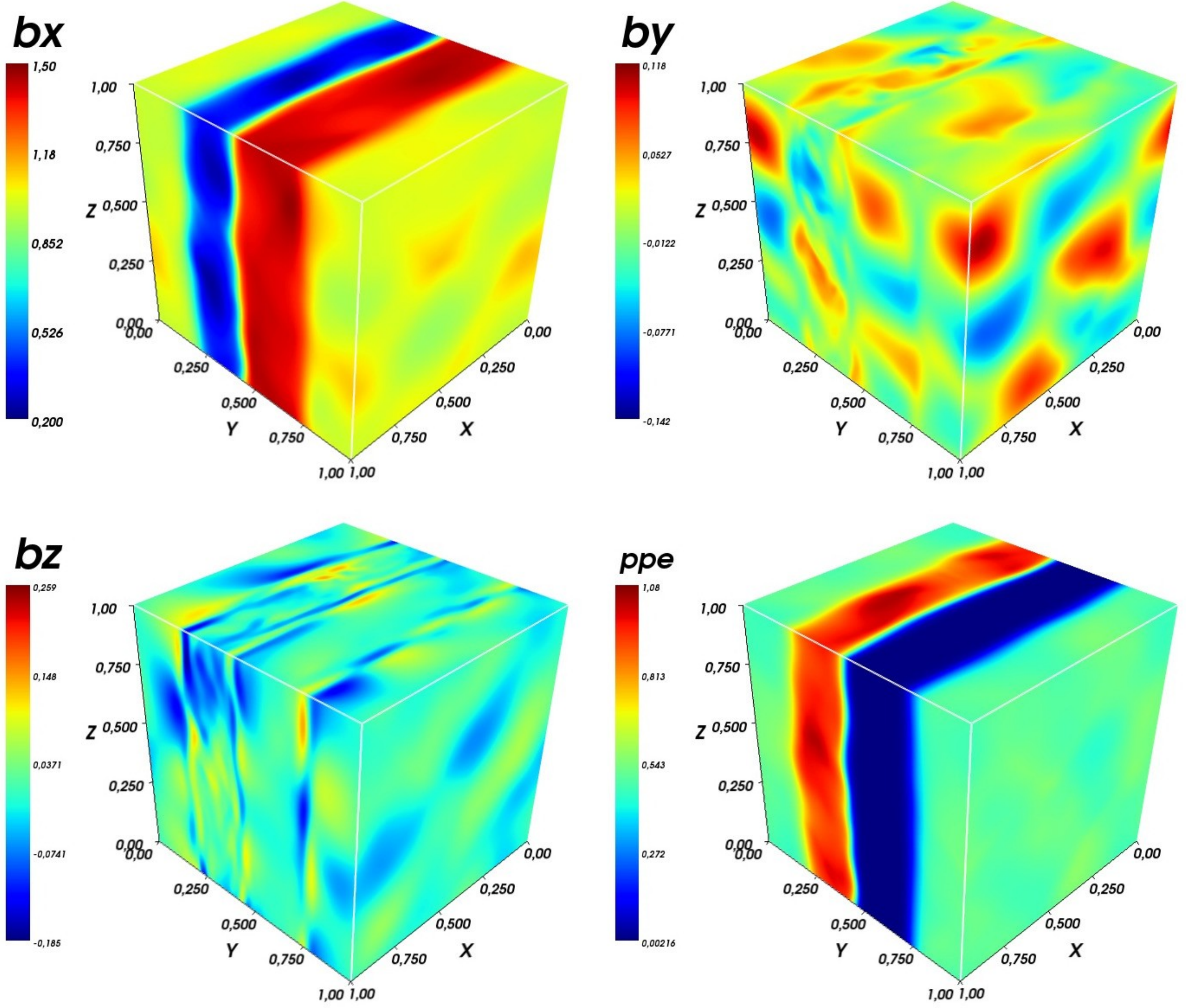
$$p = p_{\perp} = p_{\|}$$

$$p + \frac{B^2}{2\mu_0} = \text{const}$$

+

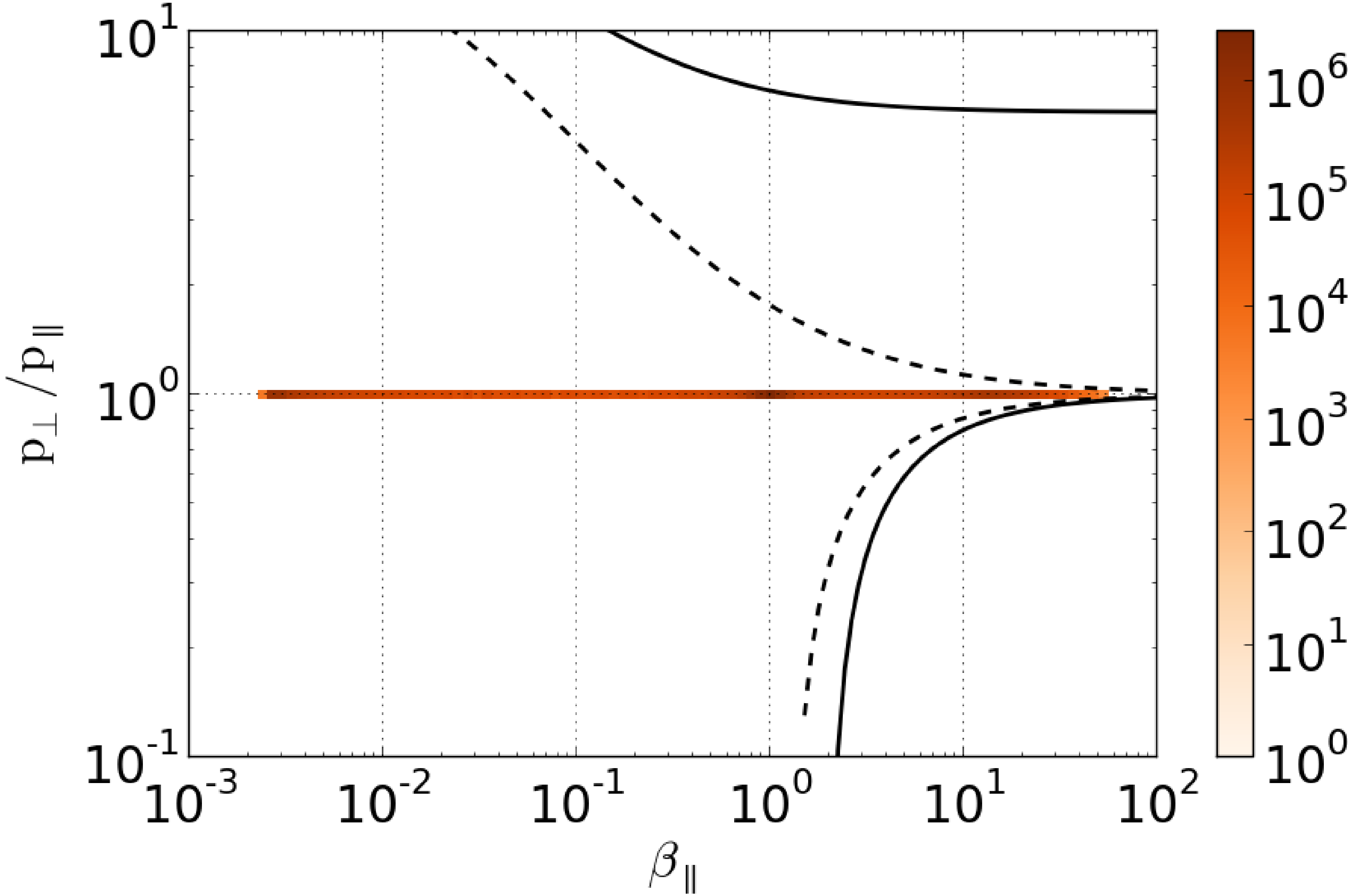
large-scale magnetic field fluctuations

Magnetic relaxation, late stage of evolution:
pressure-balance structure has survived

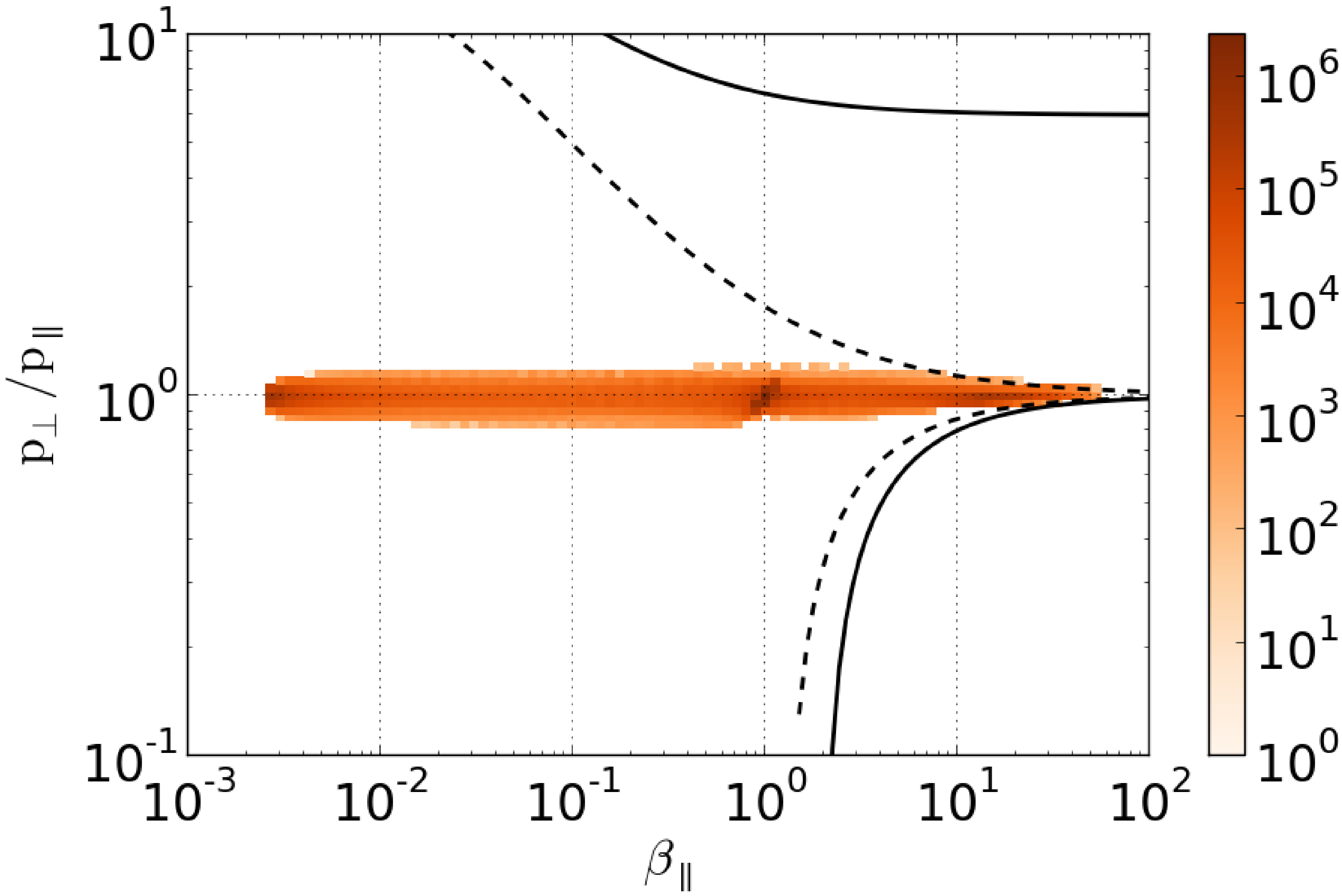


Stratified and balanced thermal and magnetic pressures help to maintain wide range of β

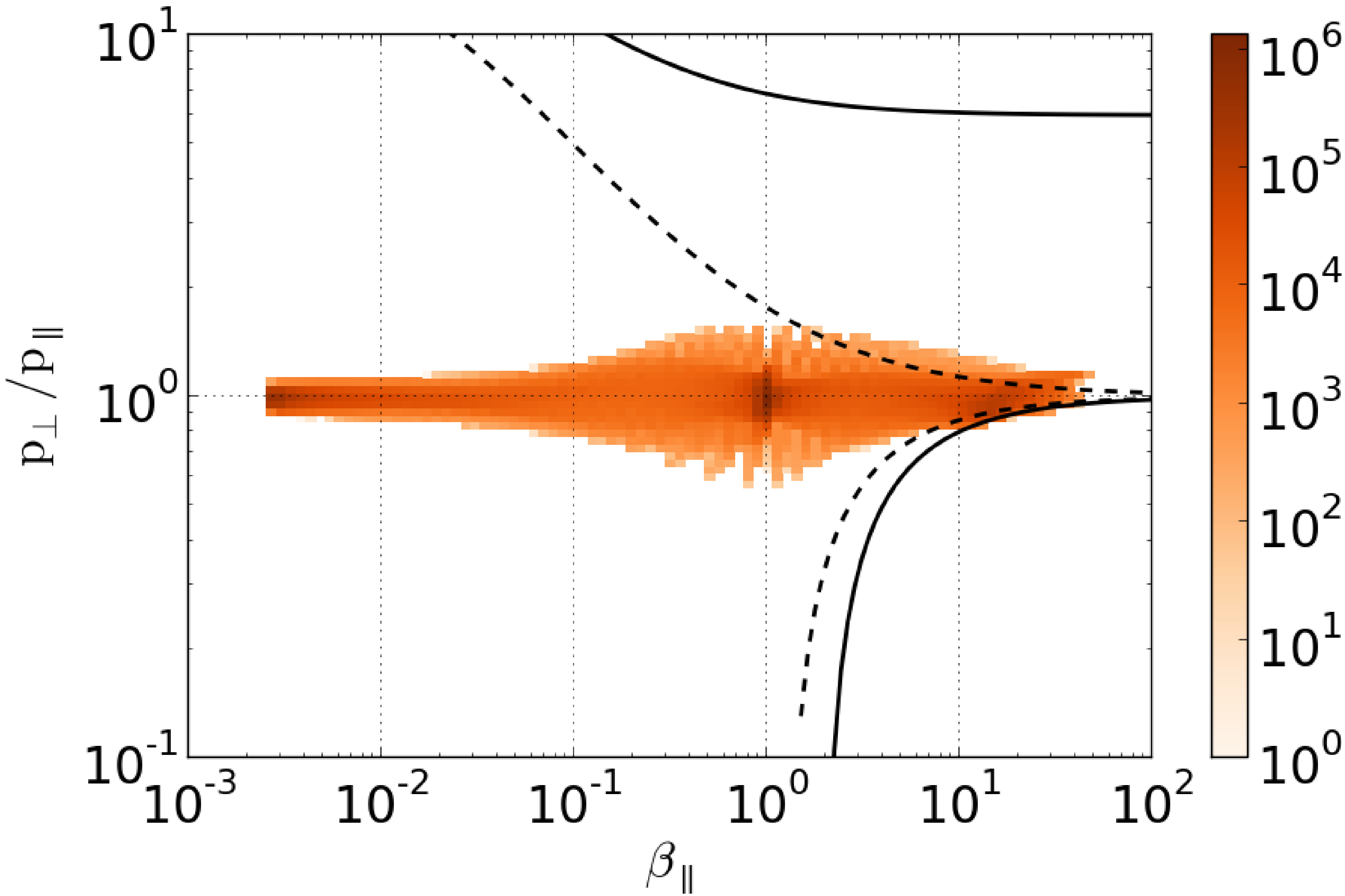
Magnetic relaxation, pressure anisotropy distribution, initial condition



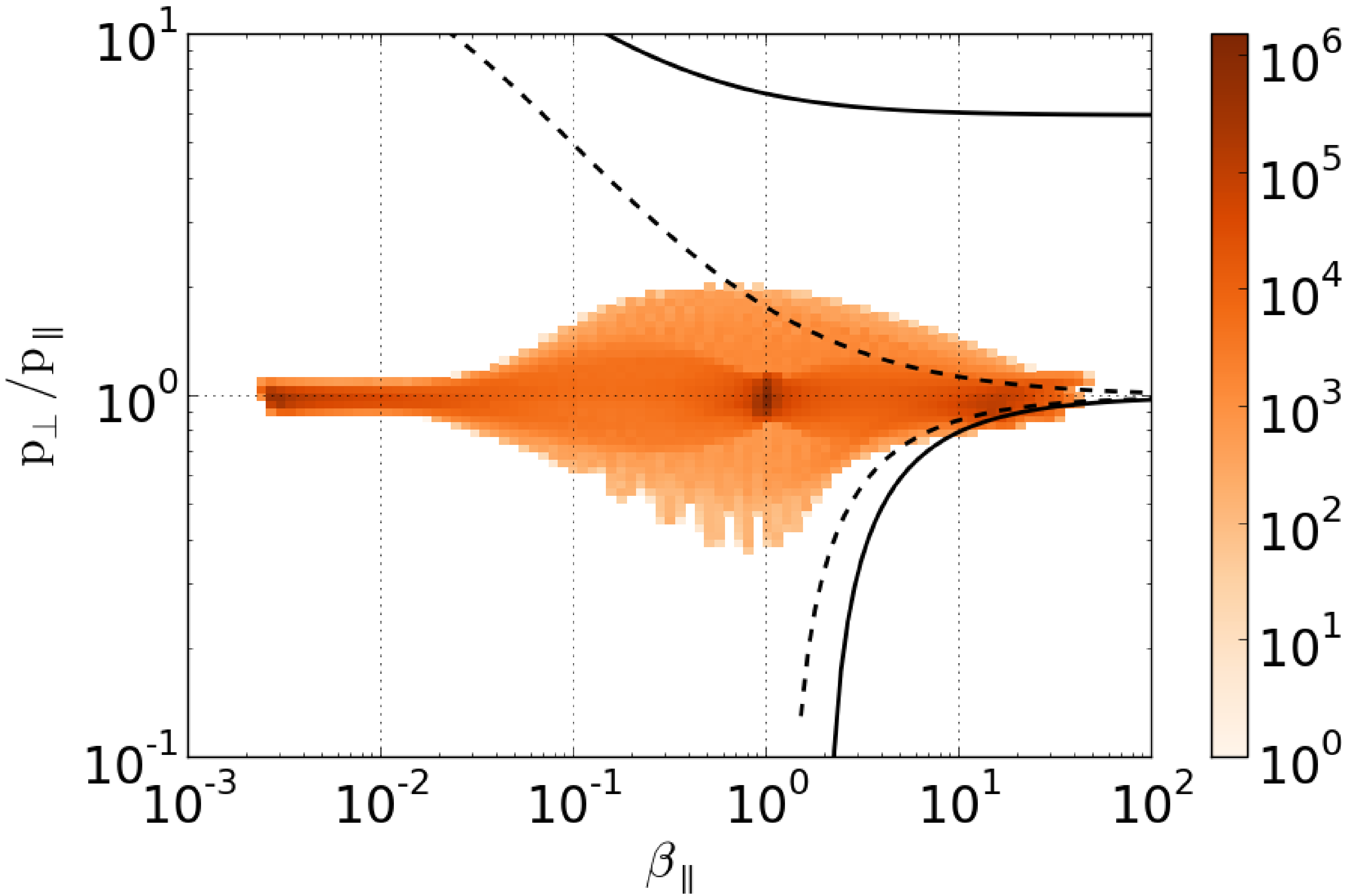
Magnetic relaxation, pressure anisotropy distribution



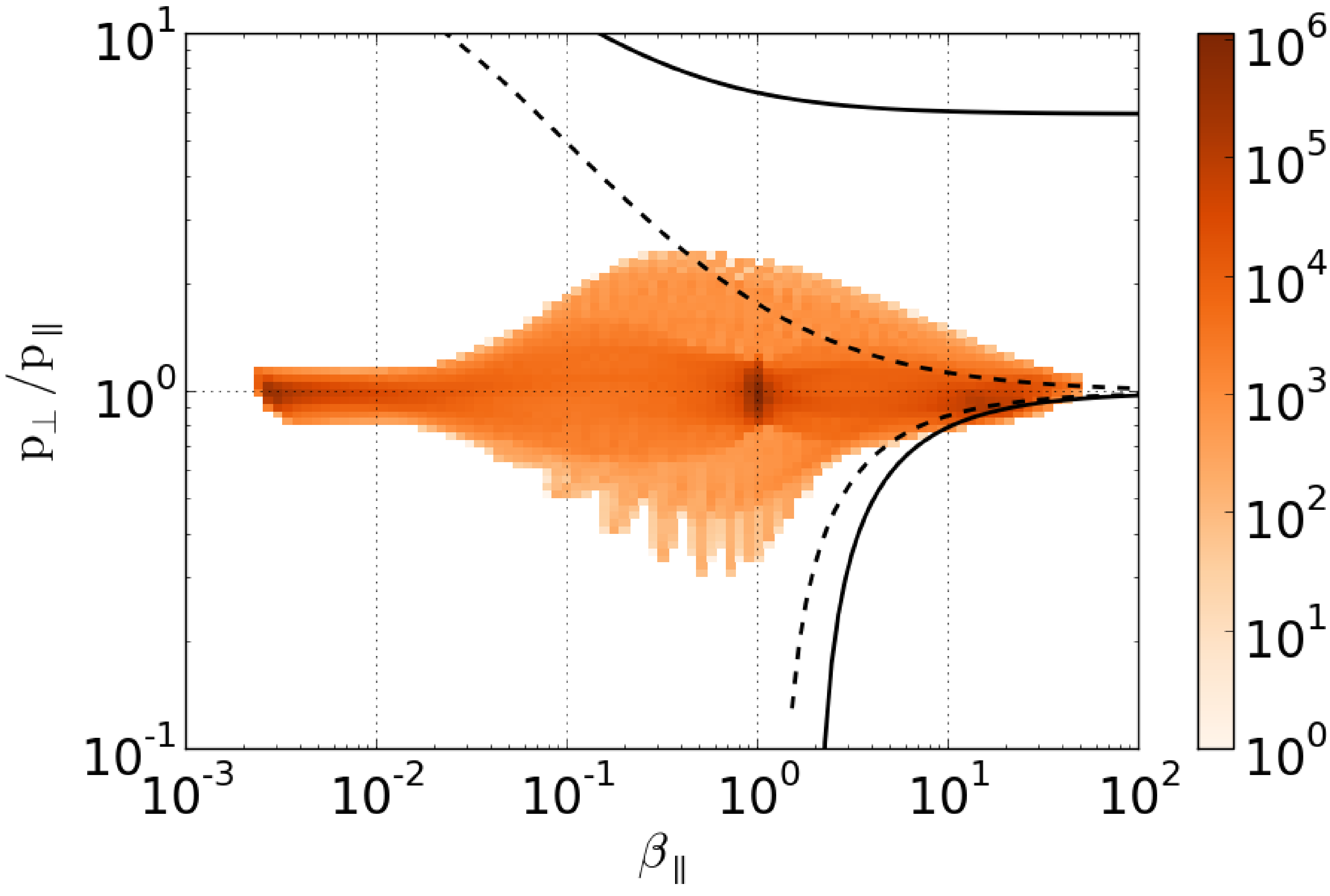
Magnetic relaxation, pressure anisotropy distribution



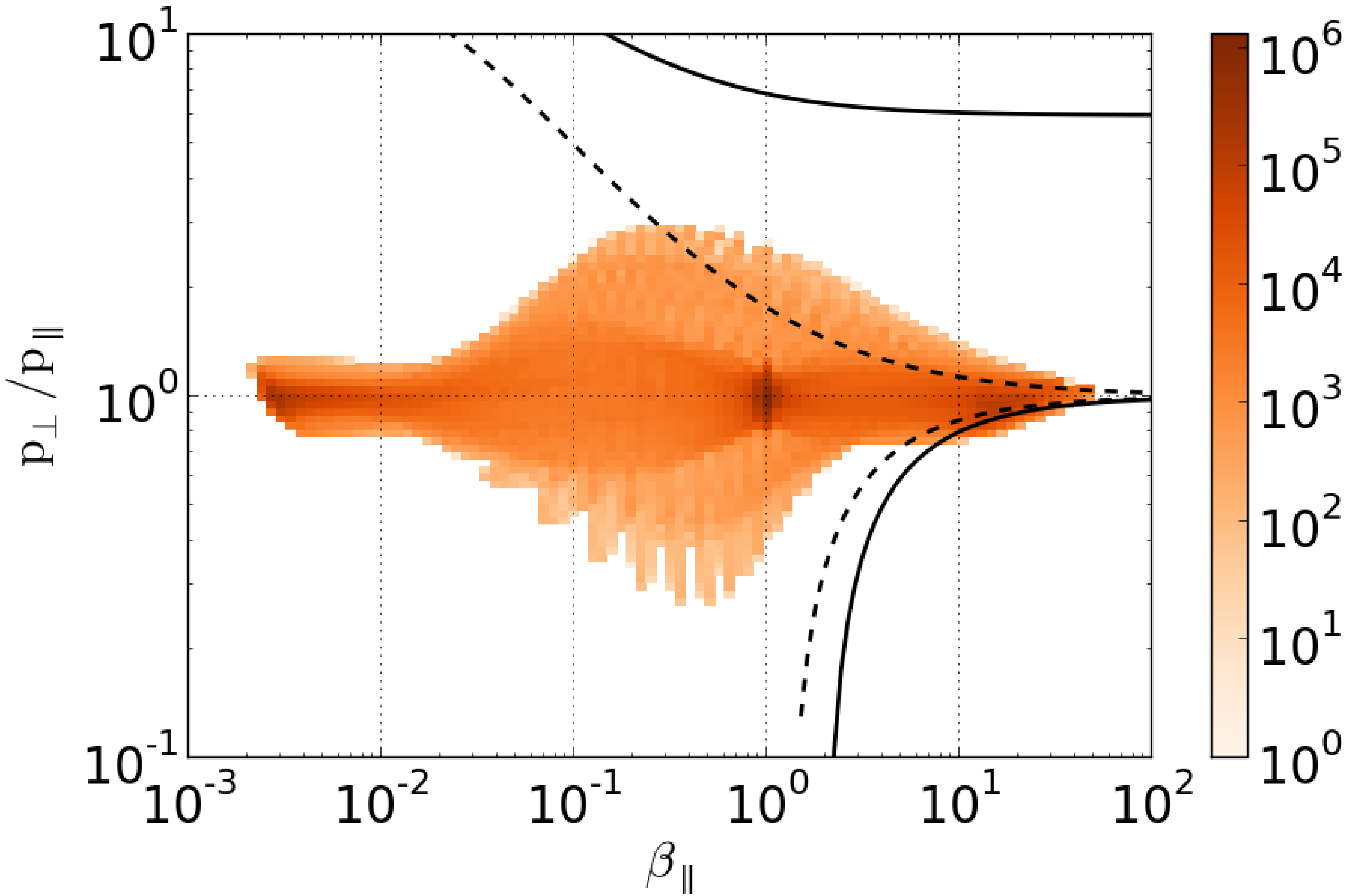
Magnetic relaxation, pressure anisotropy distribution



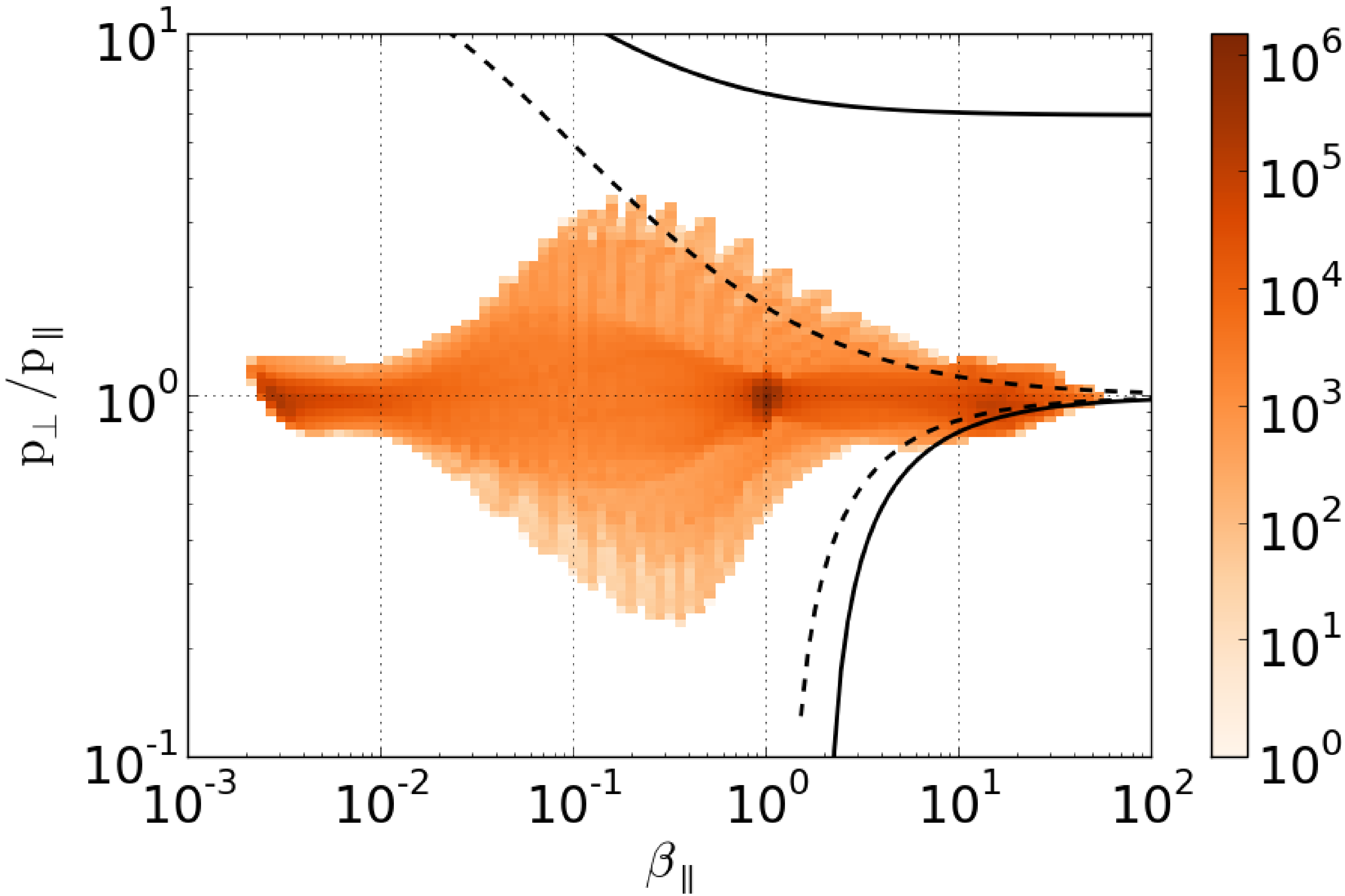
Magnetic relaxation, pressure anisotropy distribution



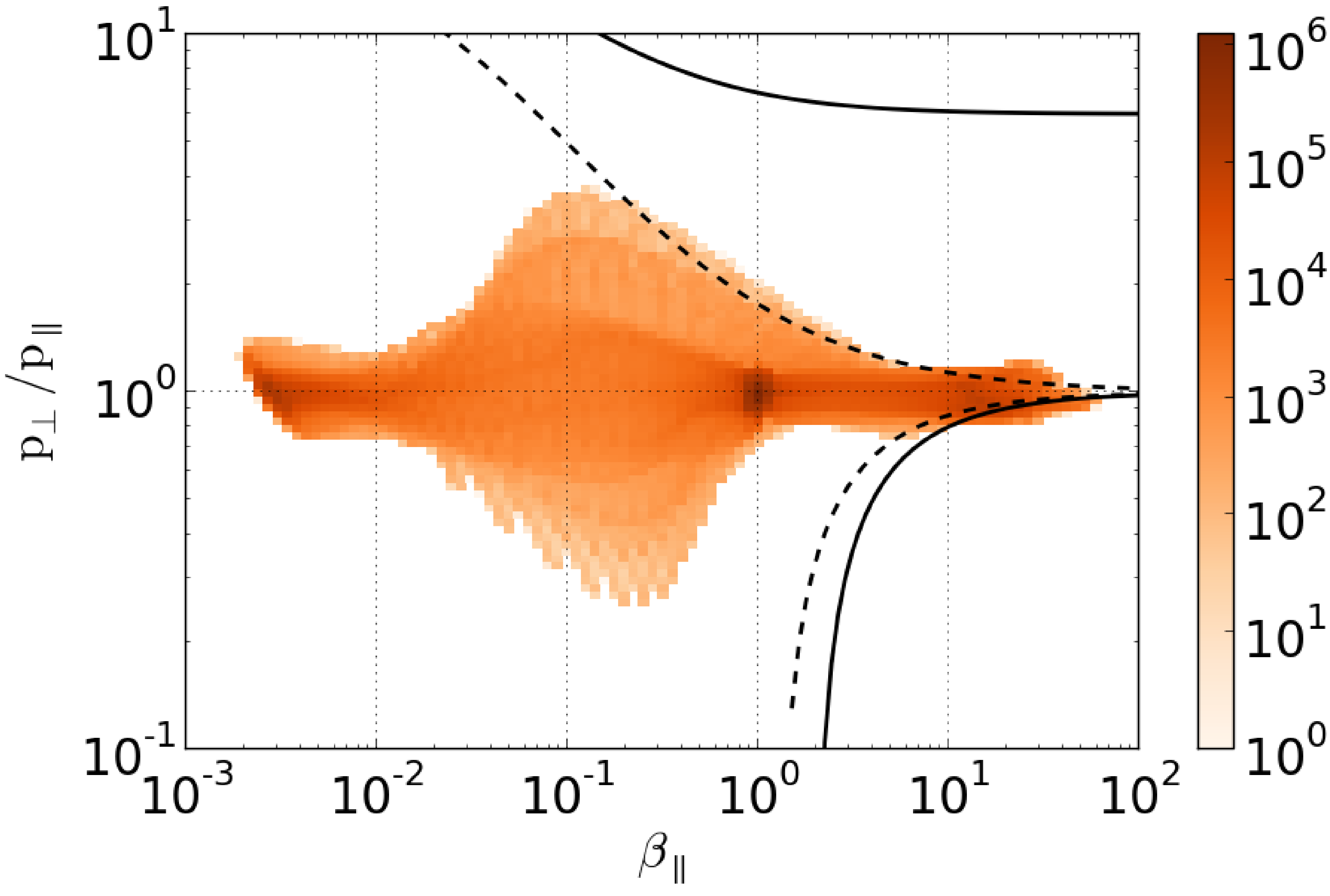
Magnetic relaxation, pressure anisotropy distribution



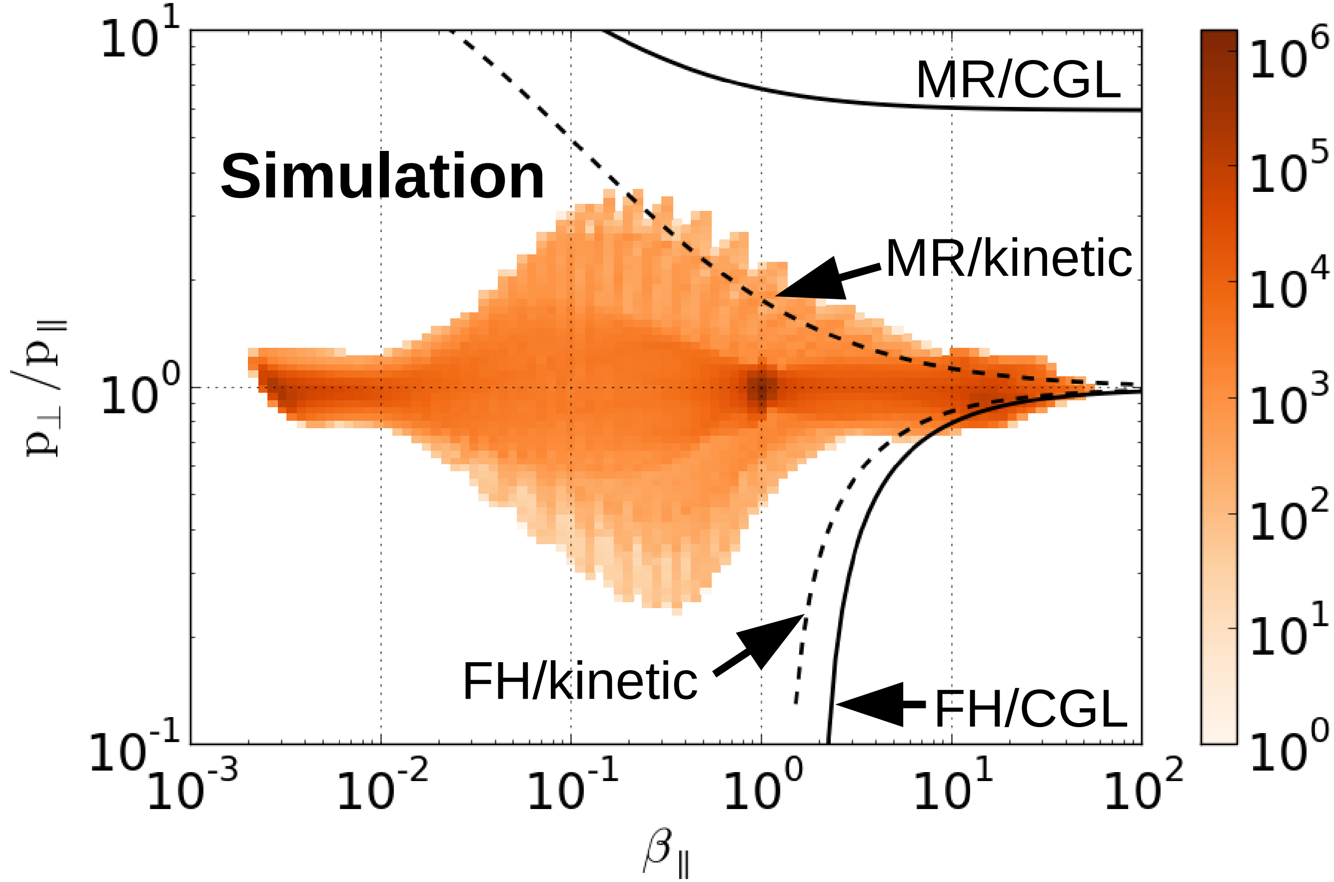
Magnetic relaxation, pressure anisotropy distribution

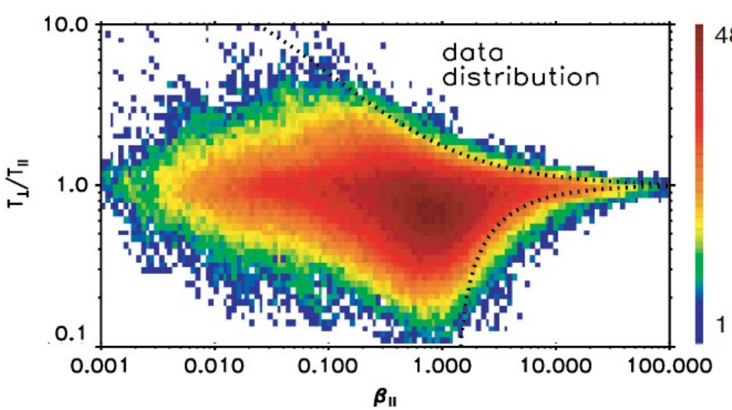


Magnetic relaxation, pressure anisotropy distribution



Magnetic relaxation in the presence of pressure-balance structure: **pressure anisotropy distribution**





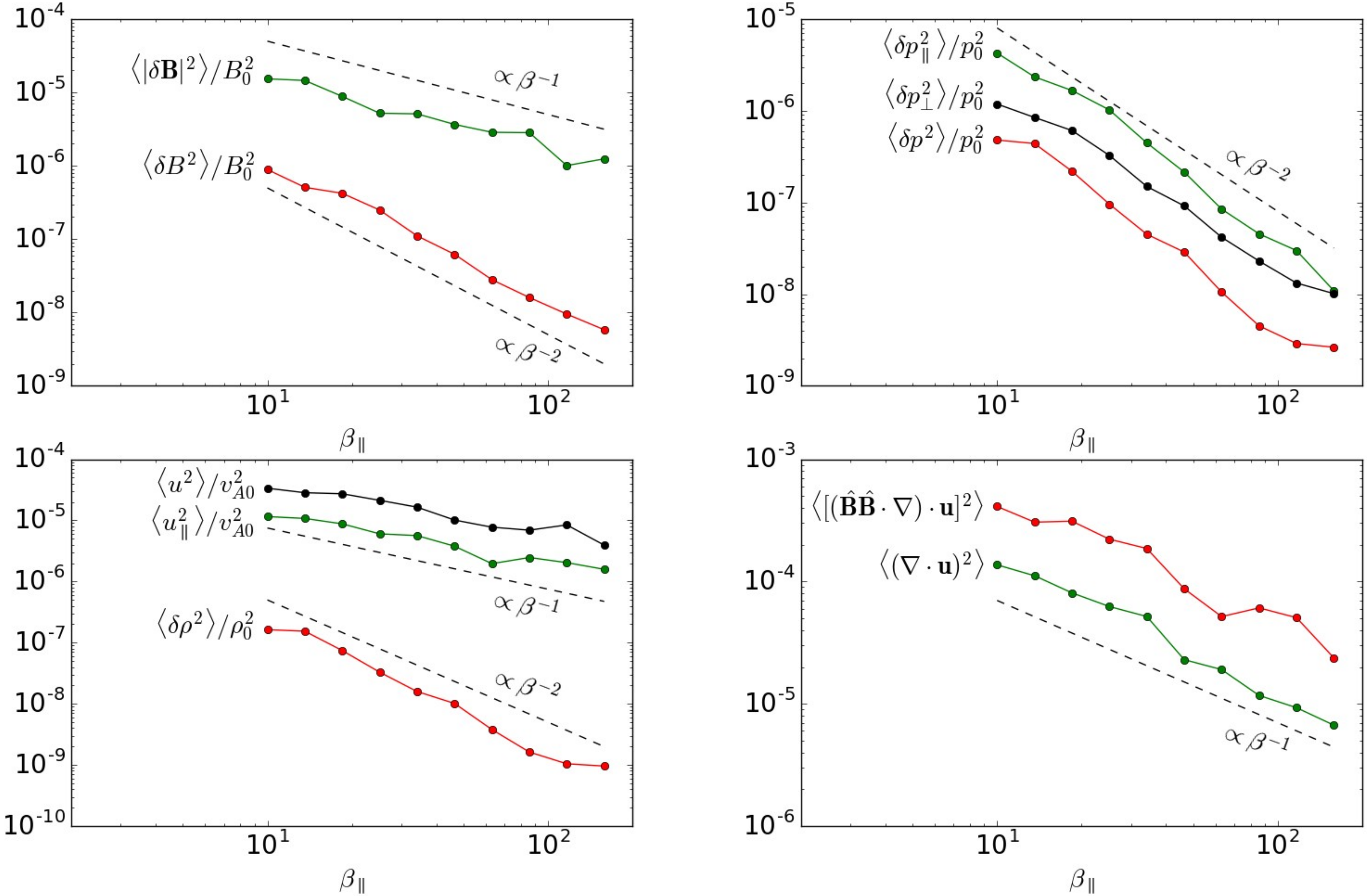
Solar wind
Bale et al., PRL, 2009

Transient pressure anisotropy distribution in MHD/CGL simulation resembles solar wind observations?

What arrests the evolution of the anisotropy distribution in high-β regime far from the mirror/firehose CGL thresholds and gives constraints similar to kinetic instabilities?

Randomly forced, then decaying flow: scalings

High-β regime, β “scans” by changing average pressure



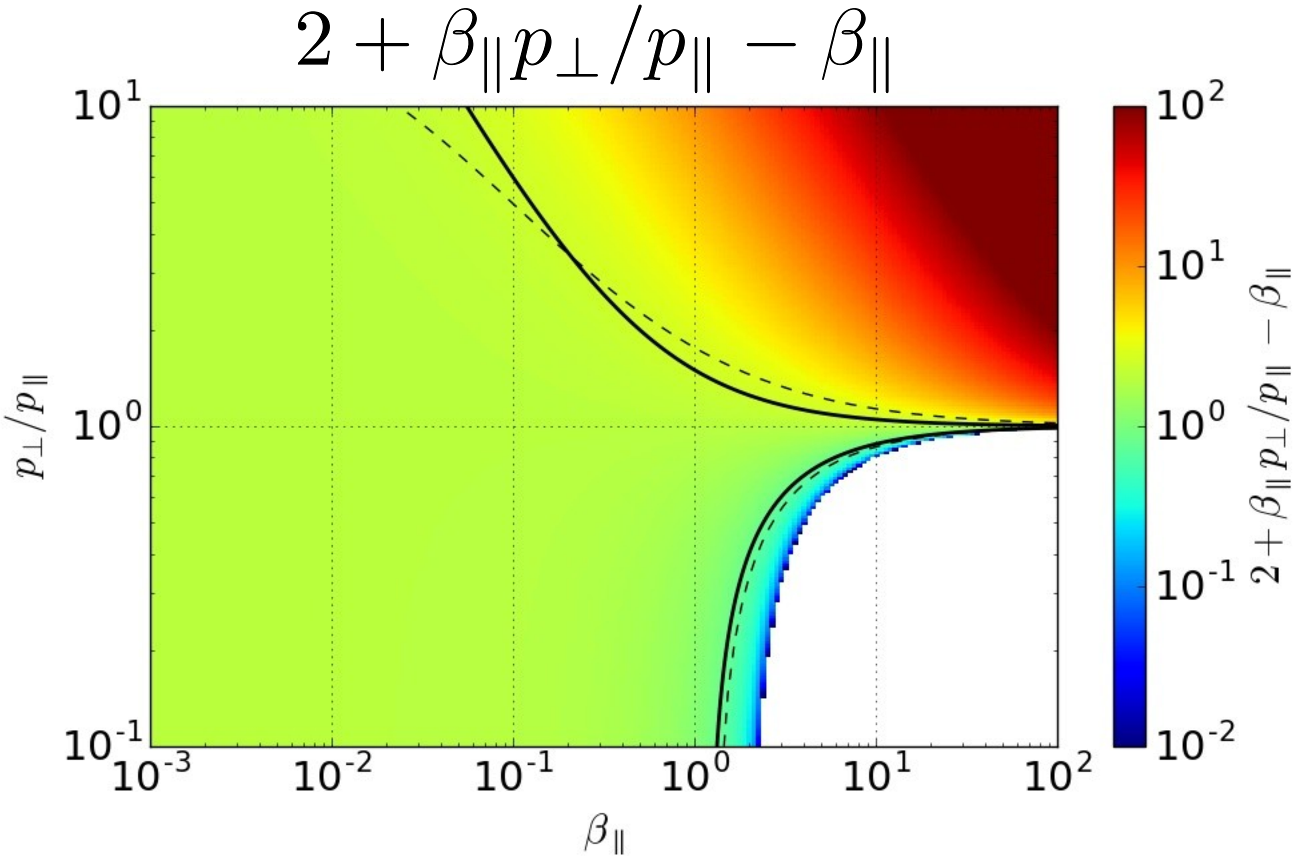
Momentum transport with pressure anisotropy

$$\rho\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\nabla_{\|}p_{\|} + \left(\frac{p_{\|}-p_{\perp}}{B}\right)\nabla_{\|}B + \ldots$$

$$-\nabla_{\perp}\left(p_{\perp}+\frac{B^2}{2\mu_0}\right) - \left(p_{\|}-p_{\perp}-\frac{B^2}{\mu_0}\right)(\hat{\mathbf{B}}\cdot\nabla)\hat{\mathbf{B}}$$

Dashed lines: kinetic MR and FH thresholds

Solid lines: constant magnitude of the magnetic tension for constant *B*



How anisotropy is regulated in the CGL case?

High-β regime, isotropic pressure in the initial condition

$\delta\rho, \delta B, \delta p_\perp, \delta p_\parallel$ are determined by $(\hat{\mathbf{B}}\hat{\mathbf{B}} \cdot \nabla) \cdot \mathbf{u} \neq 0, \nabla \cdot \mathbf{u} \neq 0$

velocity field that tries to produce $\delta p_\perp, \delta p_\parallel$ increases thermal stress

$$\nabla \cdot \mathbb{P} = \nabla_\parallel p_\parallel - \left(\frac{p_\parallel - p_\perp}{B} \right) \nabla_\parallel B + \nabla_\perp p_\perp + (p_\parallel - p_\perp)(\hat{\mathbf{B}} \cdot \nabla)\hat{\mathbf{B}}$$

that causes immediately back reaction on the velocity field

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbb{P}$$

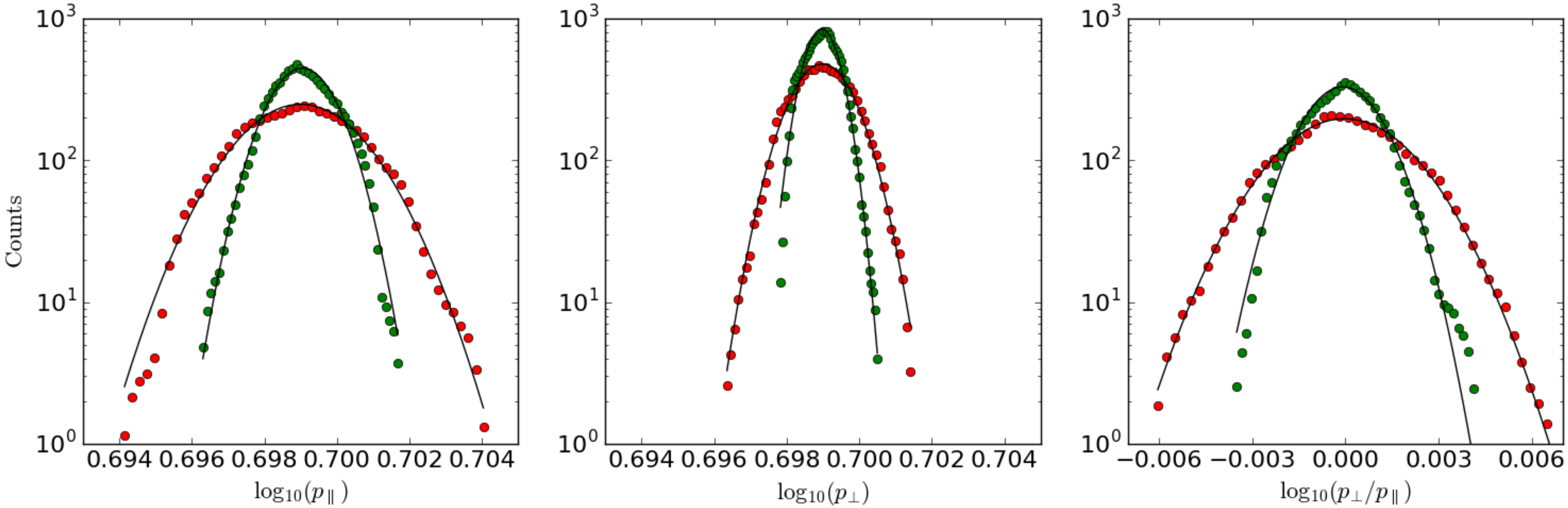
since inertial forces struggle against thermal-stress forces

If average pressure increases velocity fluctuations are forced to adjust by decreasing $(\hat{\mathbf{B}}\hat{\mathbf{B}} \cdot \nabla) \cdot \mathbf{u}, \nabla \cdot \mathbf{u}$

and thus produce smaller fluctuations of $\delta\rho, \delta B, \delta p_\perp, \delta p_\parallel$

This means that β-dependence of $p_\perp / p_\parallel$ appears

Pressure anisotropy: log-normal pdfs



- Randomly forced, then decaying flows (with constant β initially): approximately log-normal pdfs of fluctuations of pressure components and anisotropy
- It suggests an additive random process that builds up the logarithm of pressure fluctuations (also in decaying phase)
- Similar to Passot & Vazquez-Semadeni (1998) model for log-normal distributions of density fluctuations

Chew-Goldberger-Low (CGL, double-adiabatic) closure

Evolutions of pressure components

$$\frac{\mathrm{d}\log p_\perp}{\mathrm{d}t} = (\hat{\mathbf{B}}\hat{\mathbf{B}}\cdot\nabla)\cdot\mathbf{u} - 2\nabla\cdot\mathbf{u}$$

$$\frac{\mathrm{d}\log p_\parallel}{\mathrm{d}t} = -2(\hat{\mathbf{B}}\hat{\mathbf{B}}\cdot\nabla)\cdot\mathbf{u} - \nabla\cdot\mathbf{u}$$

and density and magnetic field strength

$$\frac{\mathrm{d}\log B}{\mathrm{d}t} = (\hat{\mathbf{B}}\hat{\mathbf{B}}\cdot\nabla)\cdot\mathbf{u} - \nabla\cdot\mathbf{u}$$

$$\frac{\mathrm{d}\log \rho}{\mathrm{d}t} = -\nabla\cdot\mathbf{u}$$

are determined by velocity fluctuations $(\hat{\mathbf{B}}\hat{\mathbf{B}}\cdot\nabla)\cdot\mathbf{u}, \nabla\cdot\mathbf{u}$

Connection with dynamo problem

Pressure anisotropy and magnetic field strength are related to each other in MHD-CGL approximation: regulation of pressure anisotropy is connected to constraints on the magnetic field amplification in turbulent flows

$$W = \left\langle \frac{\rho u^2}{2} + p_\perp + \frac{p_\parallel}{2} + \frac{B^2}{2} \right\rangle$$

$$I = \left\langle \frac{p_\perp}{B} \right\rangle$$

$$J = \left\langle p_\parallel^{1/3} B^{2/3} \right\rangle$$

Invariants *W, I, J* impose upper bound on the maximum magnetic energy in MHD-CGL

$$A = M_1 - M_0 \sim W/\beta$$

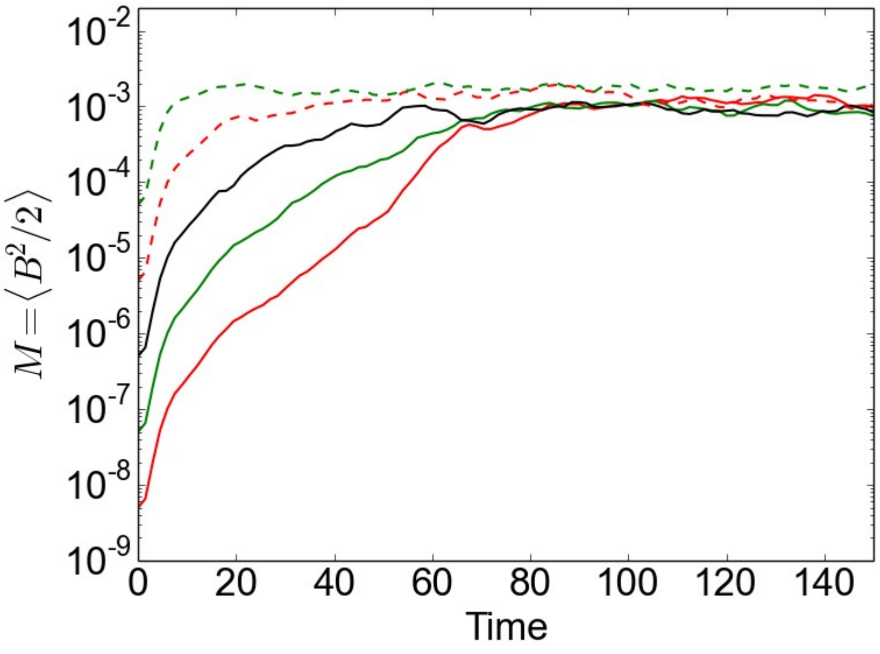
$$M = \left\langle \frac{B^2}{2} \right\rangle$$

Due to conservation of several invariants dynamo action is constrained in MHD-CGL (Helander et al., in preparation)

Connection with dynamo problem

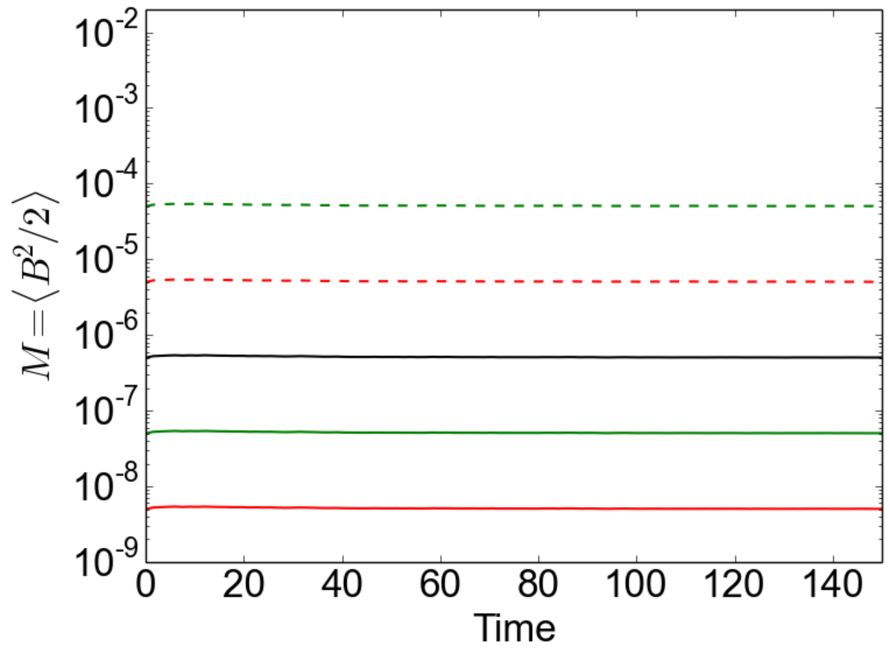
3D numerical simulations, plasma stirred up with large-scale random-phase fluctuations of velocity components

MHD (isotropic pressure)



Magnetic energy density increases

MHD-CGL



Almost constant magnetic energy density

Small-scale dynamo action requires microscopic scattering!

Conclusions

MHD-CGL approximation predicts pressure anisotropy regulation in turbulent plasma, it is confirmed by 3D numerical simulations of magnetic relaxation process

In high-β regime the regulation of pressure anisotropy is presumably implied by struggle between inertial forces and thermal stress, magnetic field is dynamically important even in high-β limit due to its presence in state equations

In MHD-CGL model pressure-anisotropy regulation is related to turbulent dynamo problem, dynamo action is also constrained

MHD-CGL pressure anisotropy pattern for magnetic relaxation process is similar to distribution observed by WIND spacecraft

Momentum transport in the presence of the pressure anisotropy

Momentum transport

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right] = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbb{P}$$

Maxwell-stress-related terms

$$\mathbf{J} \times \mathbf{B} = \frac{B^2}{\mu_0}(\hat{\mathbf{B}} \cdot \nabla)\hat{\mathbf{B}} - \frac{1}{2\mu_0} \nabla_\perp B^2$$

Thermal-stress-related terms

$$\nabla \cdot \mathbb{P} = \nabla_{||} p_{||} - \left(\frac{p_{||} - p_\perp}{B} \right) \nabla_{||} B + \nabla_\perp p_\perp$$

$$+ (p_{||} - p_\perp)(\hat{\mathbf{B}} \cdot \nabla)\hat{\mathbf{B}}$$

MHD eqs + CGL closure

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbb{P}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \nabla \cdot \mathbf{B} = 0 \qquad \mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$$

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{p_\perp}{\rho B} \right) = 0 \qquad \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{p_\parallel B^2}{\rho^3} \right) = 0$$

Conservative form of MHD-CGL eqs

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} = -\nabla \cdot \left(\rho \mathbf{u}\mathbf{u} + \mathbb{P} - \frac{\mathbf{B}\mathbf{B}}{\mu_0} + \frac{B^2}{2\mu_0}\mathbb{I} \right)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \cdot (\mathbf{u}\mathbf{B} - \mathbf{B}\mathbf{u})$$

$$\frac{\partial E}{\partial t} = -\nabla \cdot \left[\left(E + \frac{B^2}{2\mu_0} \right) \mathbf{u} - (\mathbf{u} \cdot \mathbf{B})\mathbf{B} + \mathbb{P} \cdot \mathbf{u} \right]$$

$$\frac{\partial (p_\perp / B)}{\partial t} = -\nabla \cdot (\mathbf{u}\, p_\perp / B) \qquad \mathbb{P} = p_\perp \delta_{ij} + (p_\parallel - p_\perp) \frac{B_i B_j}{B^2}$$

$$E = \rho u^2/2 + p/(\gamma - 1) + B^2/2\mu_0 \qquad p = (2p_\perp + p_\parallel)/3$$