Collisions of two Alfvénic wave packets: beyond the Moffatt & Parker problem

Oreste Pezzi
oreste.pezzi@fis.unical.it

S. Servidio, T.N. Parashar, C.V. Vasconez, Y. Yang, F. Valentini, F. Malara, W.H. Matthaeus and P. Veltri

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Outline

- **Introduction**
    - Historical considerations on the problem... Moffatt & Parker scenario and more recent works

- **Initial configuration & models**
    - Dimensionality issue

- **“Global” analysis … beyond the Moffatt & Parker scenario**
    - Cross helicity vs generalized cross helicity
    - Current, compressive and vortical structures

- **“Novel” features**
    - Anisotropy/agyrotropic production and non-Maxwellian features
    - Weakly non-linear coupling vs turbulence
    - KAW like correlations

- **Conclusions**

Introduction: an historical problem

Ideal Incompressible MHD theory

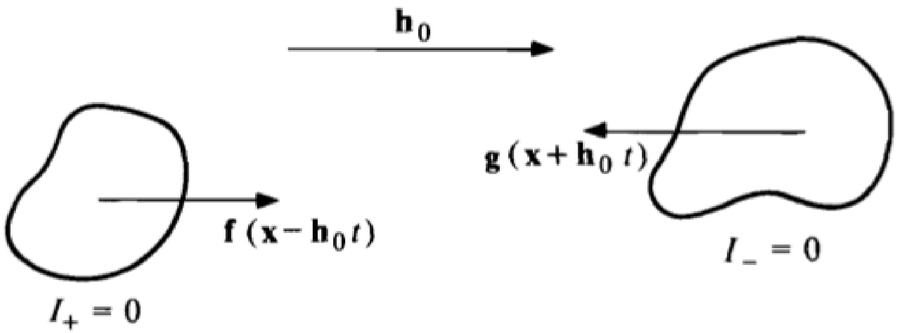


Fig. 10.1 Disturbances represented by the functions $\mathbf{f}(\mathbf{x}-\mathbf{h}_0 t)$ and $\mathbf{g}(\mathbf{x}+\mathbf{h}_0 t)$ interact only while they overlap in $\mathbf{x}$-space. The time-scale characteristic of non-linear interaction of disturbances of length-scale L is evidently at most of order L/h_0.

[H.K. Moffatt, *Field Generation in Electrically Conducting Fluids*, 1978]

"... The spatial structure of each disturbance will however presumably be modified by the interaction. The nature of this modification presents an intriguing problem that does not appear yet to have been studied." **[Moffatt, p248.]**

1) Wave packets interact ONLY during their overlapping;
2) Before the overlapping, each packet propagates without dispersion and the initial wave form is preserved. During the overlapping dispersion and nonlinear coupling may occur.

[E.N. Parker, *Cosmical magnetic fields*, 1979]

More recently, the weakly nonlinear wave-wave coupling of Alfvén waves **[Kraichnan, 1965]** has been revisited using experiments, linear theory and gyrokinetic simulations and was suggested to be an important mechanism to trigger turbulent cascade in astrophysical plasmas.

[Howes et al, POP 2013, I → IV]
[Drake, POP 2016]

Models

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla}\cdot(\rho \boldsymbol{u}), \quad (1)$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} + \frac{1}{\rho}[(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}] - \frac{\beta}{2\rho}\boldsymbol{\nabla}(\rho T) - \nu_4\nabla^4\boldsymbol{u}, \quad (2)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times\left[\boldsymbol{u}\times\boldsymbol{B} - \frac{\epsilon_h}{\rho}(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}\right] - \eta_4\nabla^4\boldsymbol{B}, \quad (3)$$

$$\frac{\partial T}{\partial t} = -(\boldsymbol{v}\cdot\boldsymbol{\nabla})T - (\gamma-1)(\boldsymbol{\nabla}\cdot\boldsymbol{v})T - \chi_4\nabla^4 T, \quad (4)$$

3 models
4 codes

$\varepsilon_H = 0$ ⟶ MHD (spectral)
$\varepsilon_H \neq 0$ ⟶ HMHD (spectral)
$\gamma = 5/3$

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} + (\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\frac{\partial f}{\partial \mathbf{v}} = 0$$

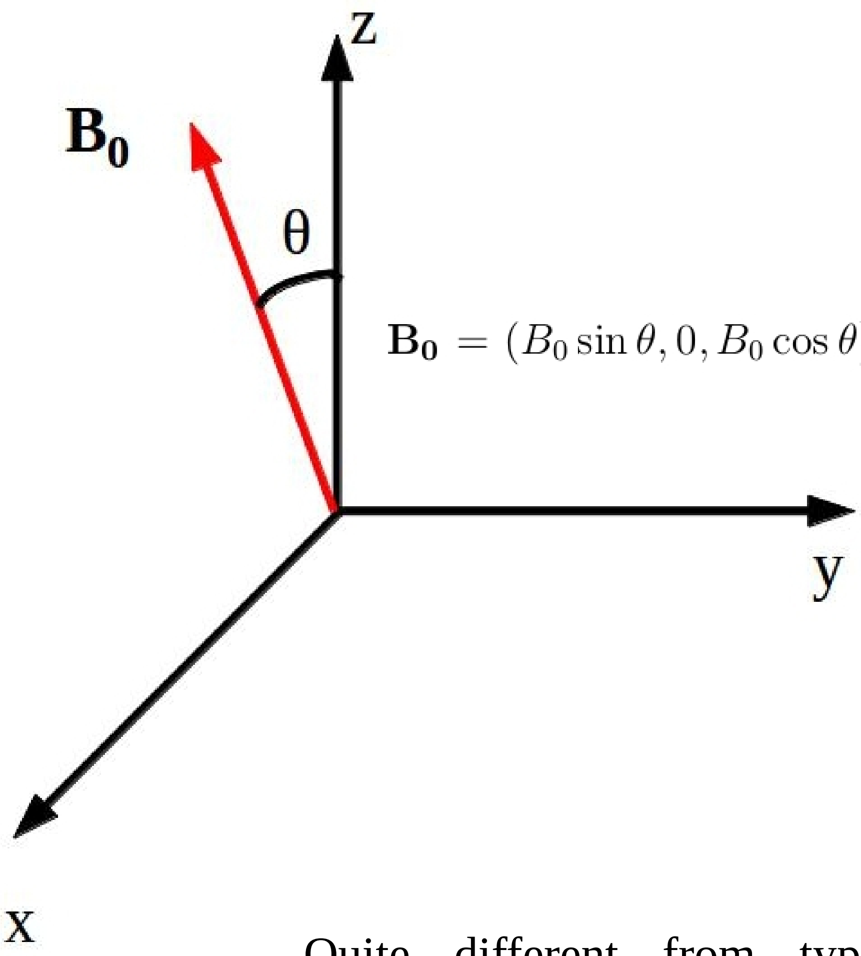
- Hybrid model (fluid, isothermal electrons)
- Ohm's generalized law
- No resistivity/viscosity

… PIC and Vlasov …

400 ppc

51^3 grid points

Initial configuration



$\mathbf{B_0} = (B_0 \sin\theta, 0, B_0 \cos\theta)$, being $\theta = 6°$

2.5D SIM

a) Wave vectors in the x-y plane
b) B_{0x} drives the packets motion

$\beta_i = 2v_{th,i}^2/v_A^2 = 0.5$

$[L_x, L_y] = [256, 64]d_i$

$[N_x, N_y] = [1024, 256]$

M_s=0.4

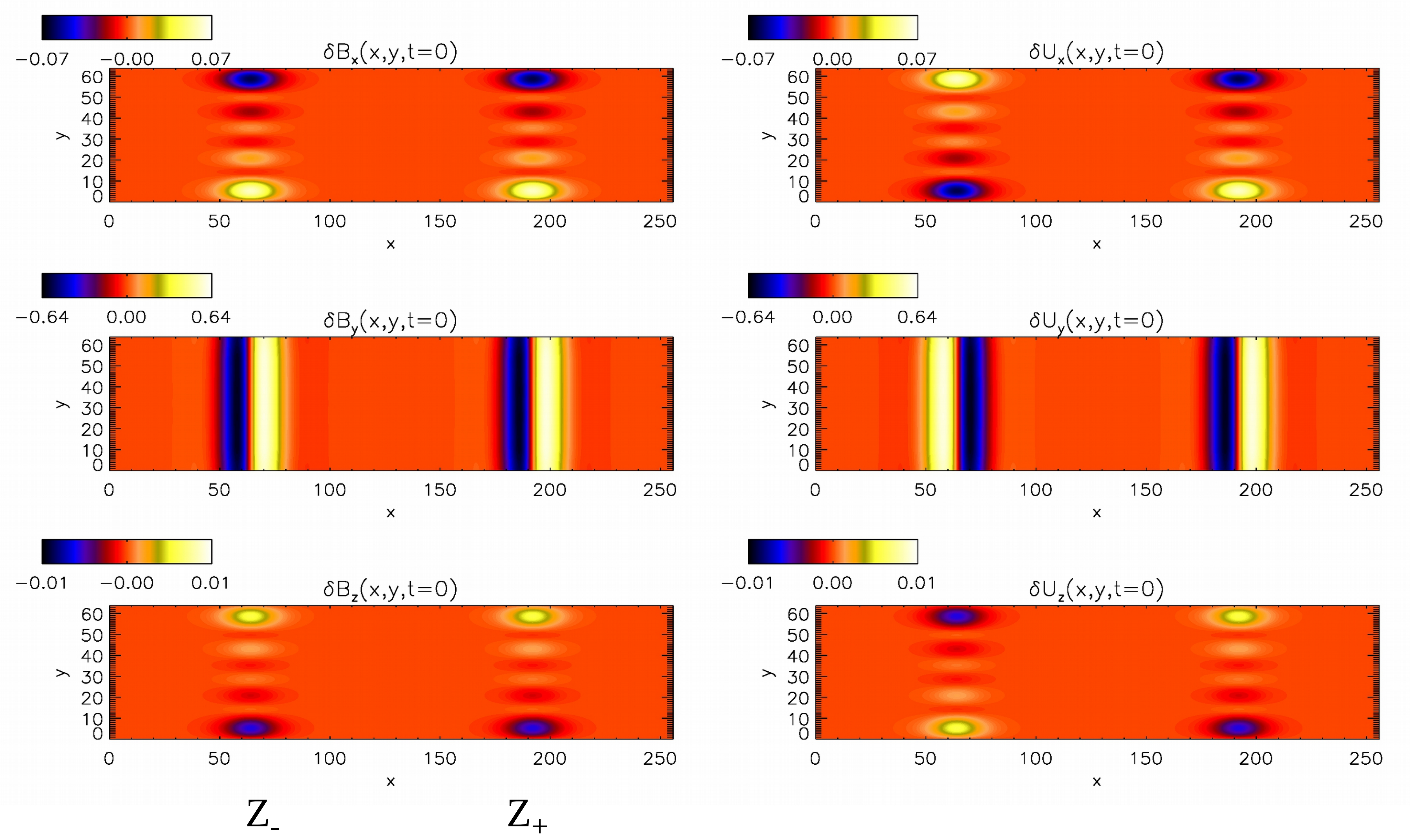
Quite different from typical coronal values. Future works will extend this analysis to lower β_i values ...

Time, spatial coordinates and velocities are scaled to Ω^{-1}_{ci}, d_i and v_A.

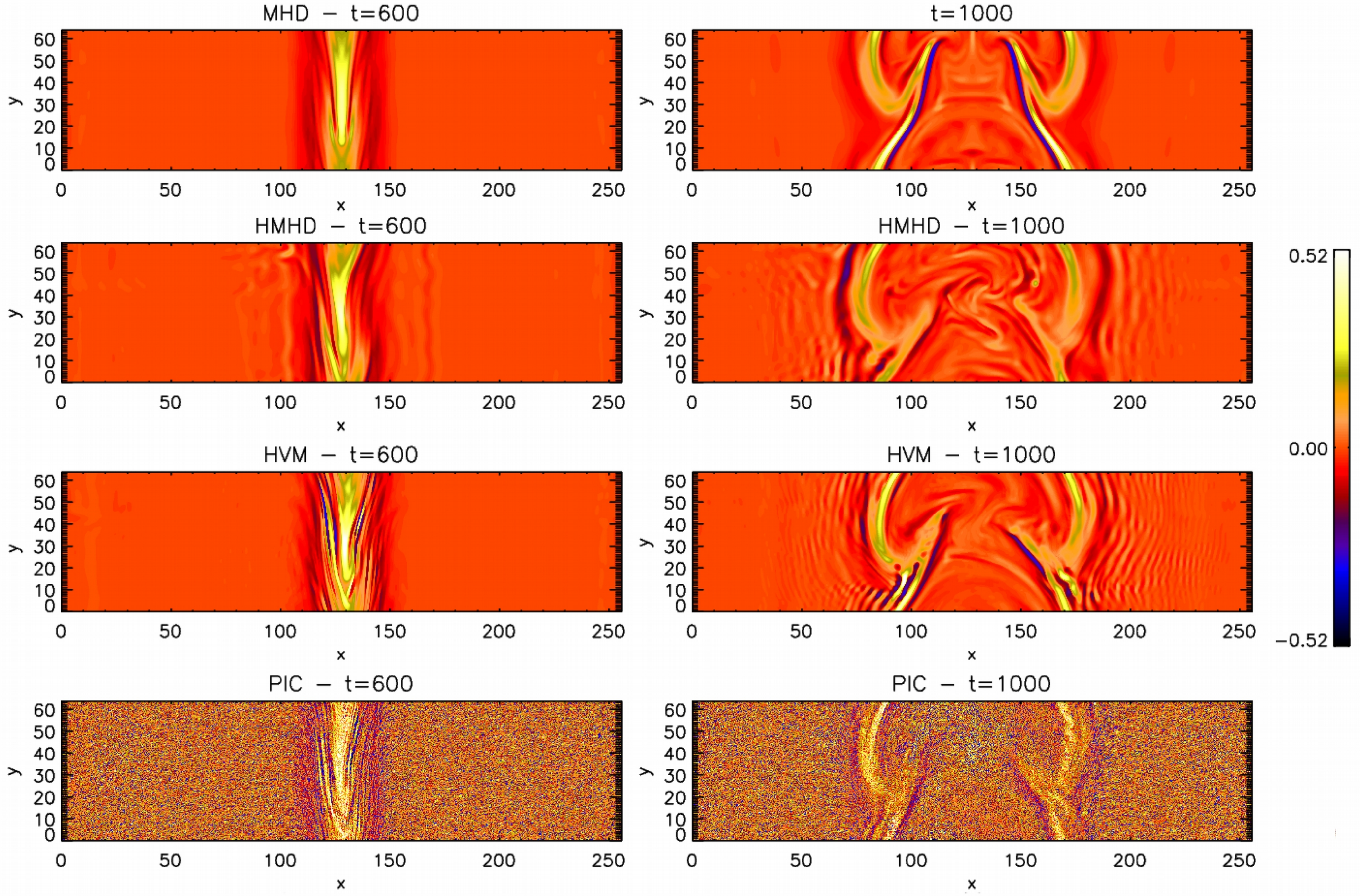
Initial configuration

$\boldsymbol{\delta B \cdot B_0} = 0$; $\mathbf{B_T} \neq \text{cost}$

$\langle \delta B/B_0 \rangle = 0.2$



Let's start …



1) Some differences between the MHD and the other runs (wave-packets spread also before the collision, smaller scales are produced, extra-ripples are present before the main wave-packet after the collision);
2) Some structures remain after the collision in the domain center;
3) PIC simulation is a bit noisy...

“Global” analysis (I)

$$\sigma_c = (e^+ - e^-)/(e^+ + e^-) = 2e^c/(e^v + e^b)$$

$$\sigma_r = (e^v - e^b)/(e^v + e^b) = 2e^r/(e^+ + e^-)$$

1) Cross-helicity production due to dispersive/kinetic effects
2) Exchange of flow vs magnetic energy during the overlapping

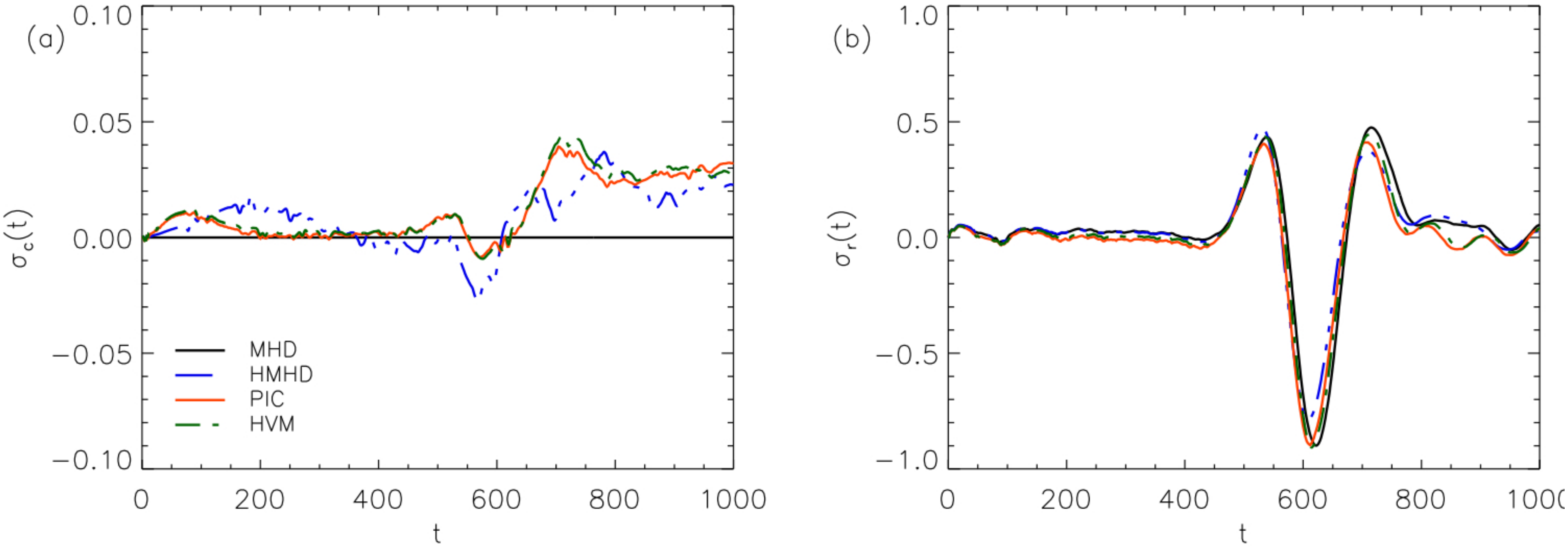
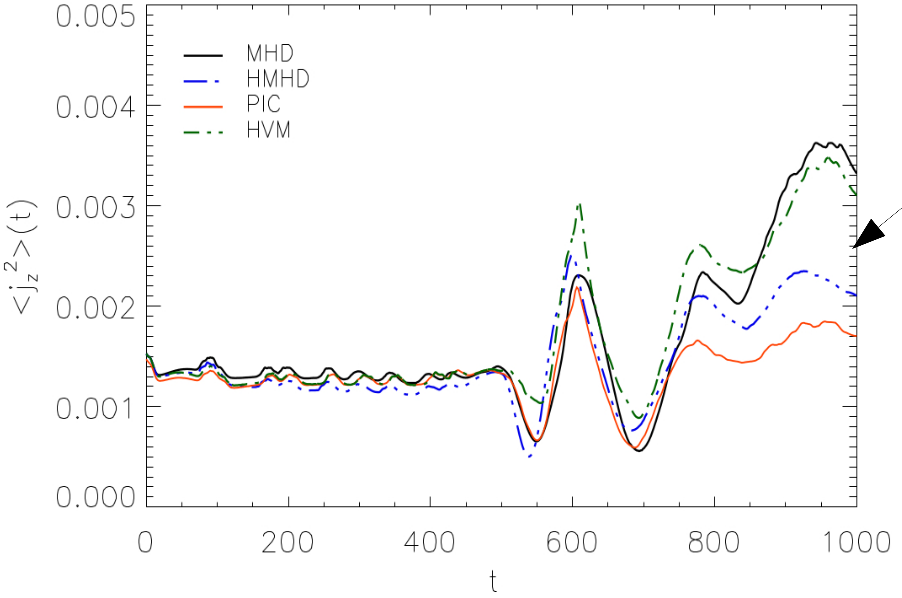
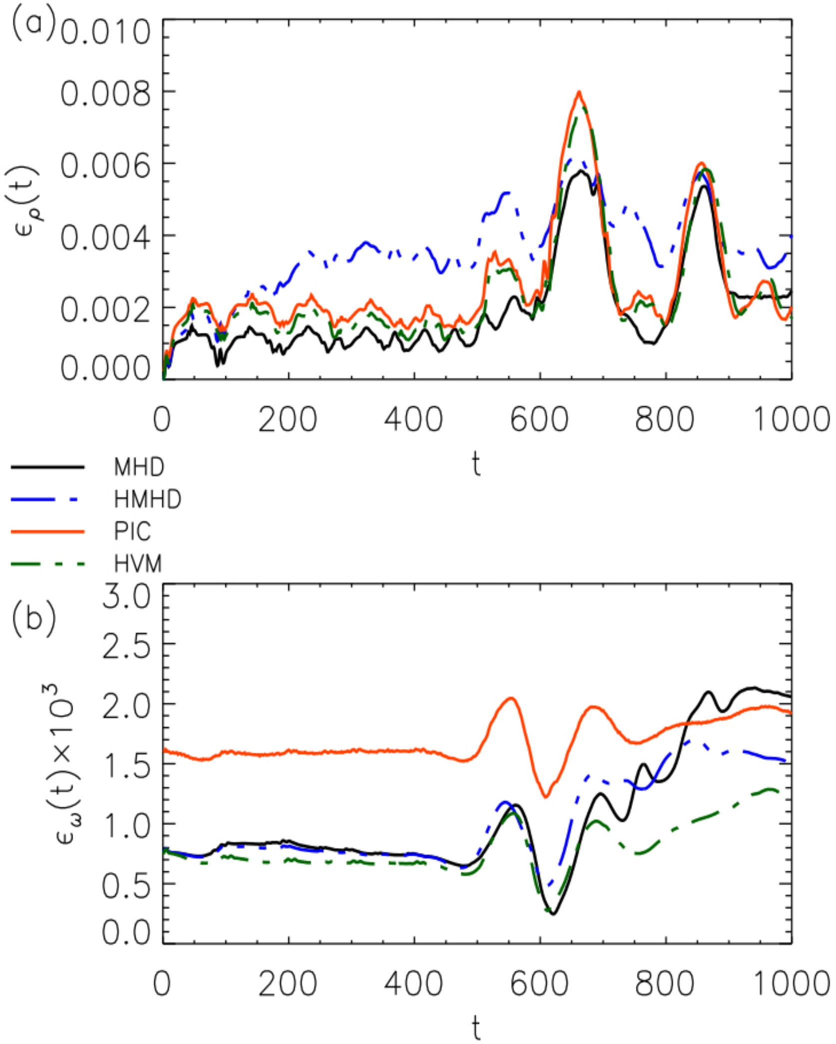


FIG. 4: Temporal evolution of the normalized cross helicity $\sigma_c(t)$ (a) and the normalized residual energy $\sigma_r(t)$ (b). In each panel black, blue, red and green lines indicate the MHD, HMHD, PIC and HVM simulations respectively.

“Global” analysis (II)

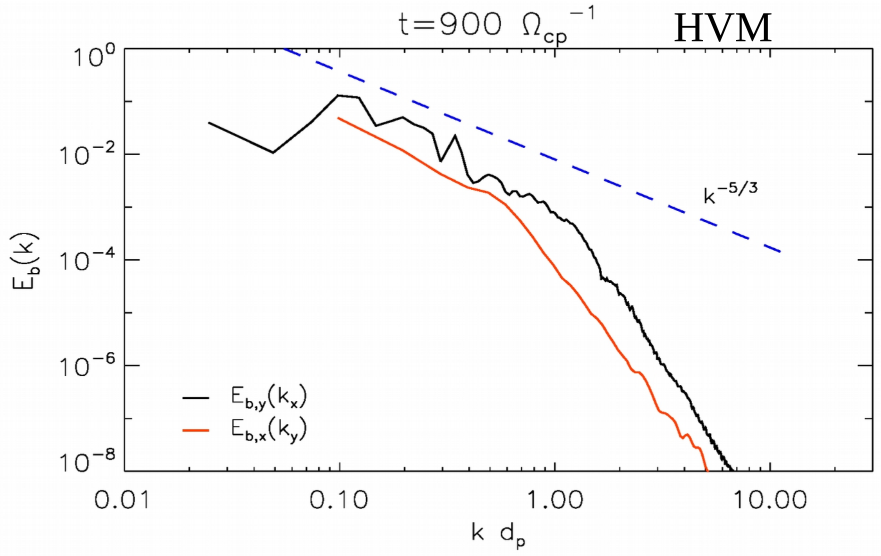


Current structures persist after the collision



Compressive and vortical structures increase after the collision.

Spectra become isotropic after the collision...

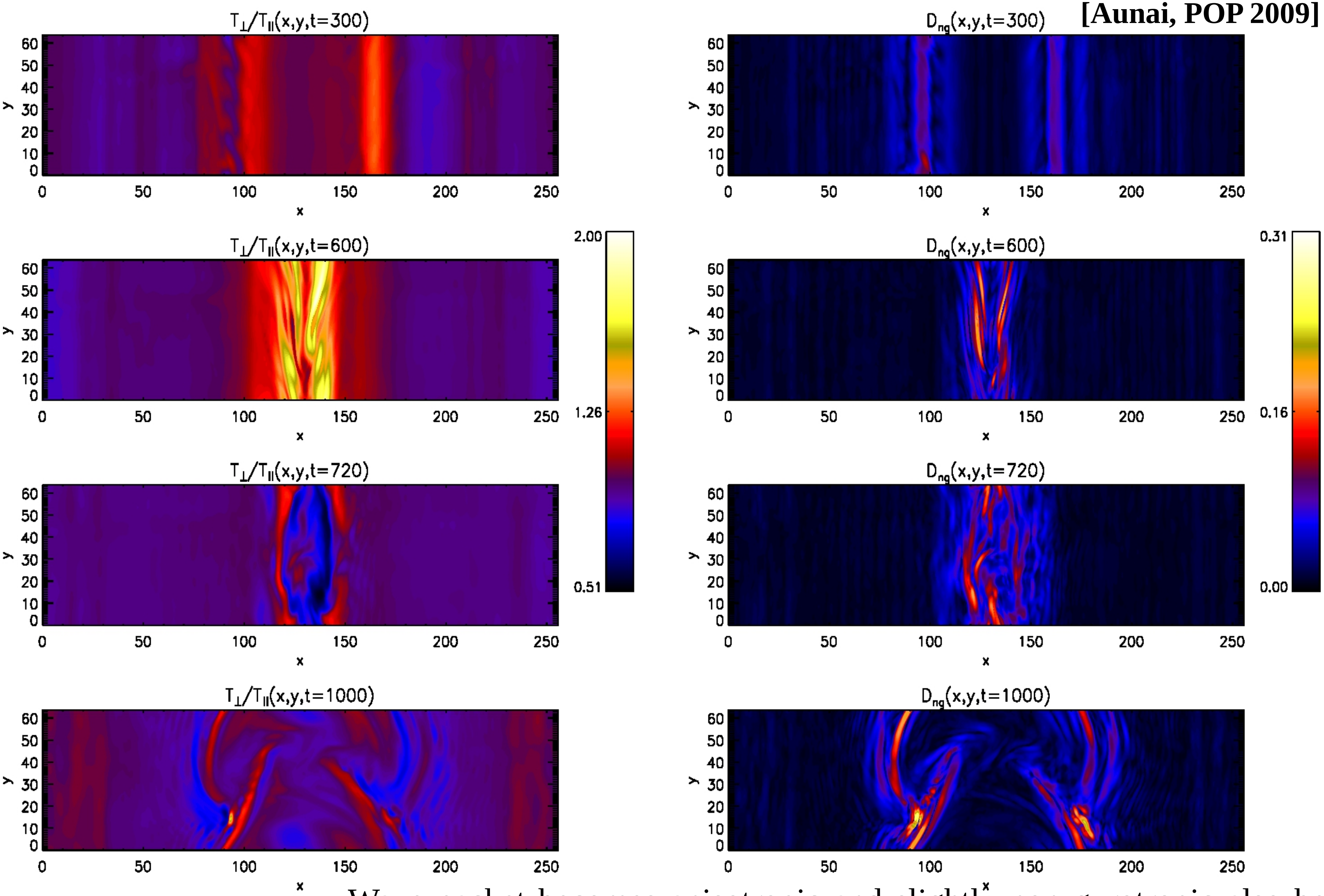


Partial summary

1) The general scenario described by **Moffatt & Parker is quite well confirmed** in the MHD case:
   1) Wave packets strongly modify their structure during the over-lapping;
   2) Small scales are in generally produced during the interaction.

2) Other features can be appreciated:
   1) When one moves to more realistic models, **dispersive/kinetic effects can play an important role**.
   2) Current structures as well as weak compressive/vortical structures persists (and also slightly increaes) after the collision and spectra tend to become isotropic.
   3) These may be signatures of the presence of some structures which remains at the center of the domain … chewing-gum modes?

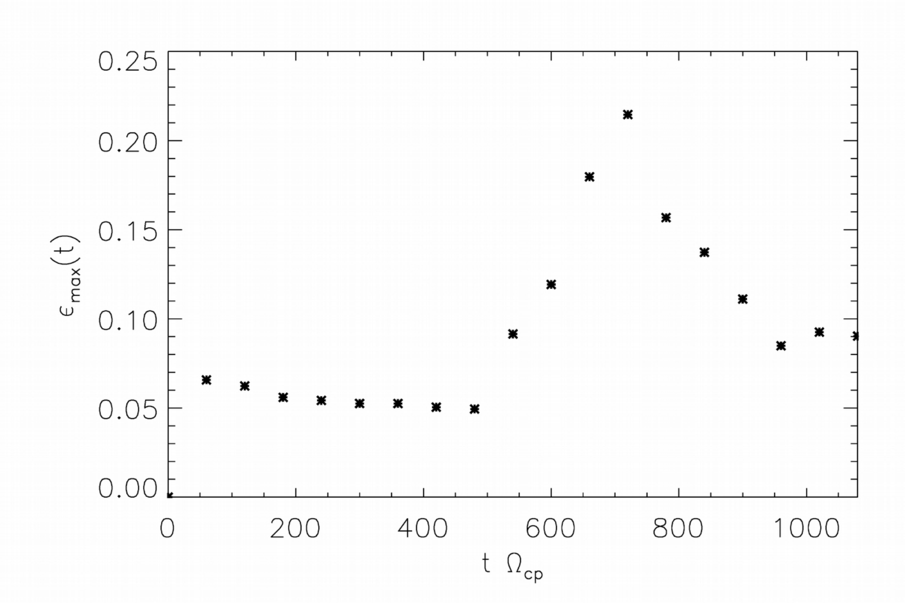
Kinetic physics: anisotropy and nongyrotropy



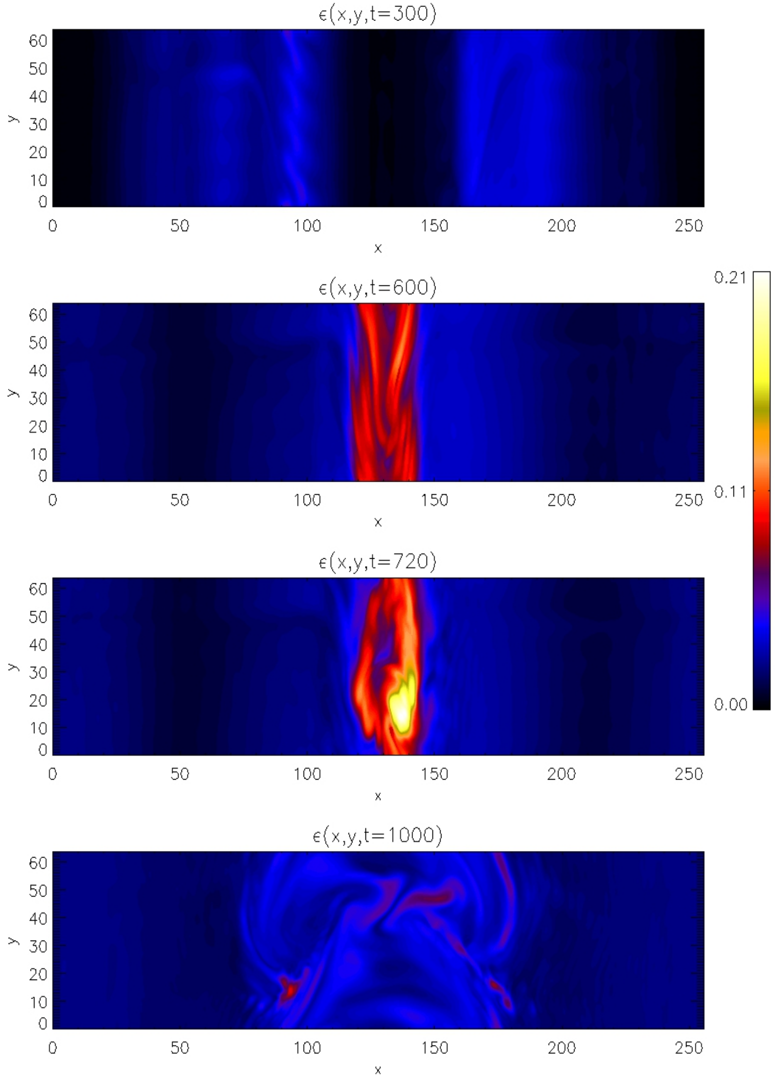
[Aunai, POP 2009]

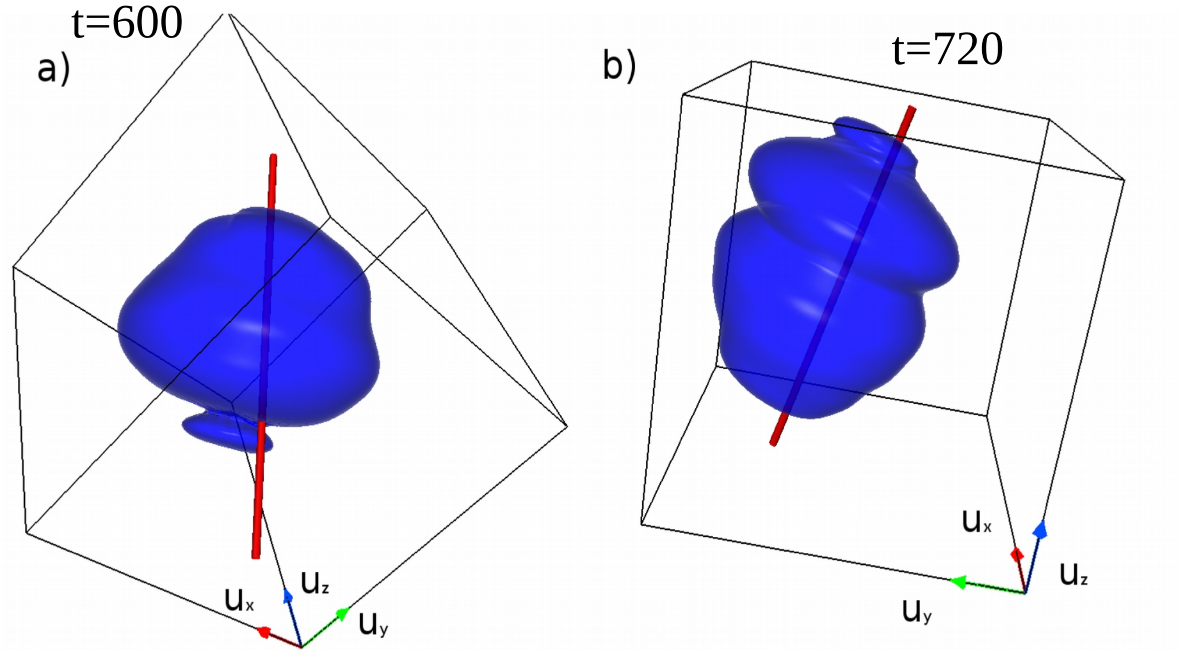
Wave packet becomes anisotropic and slightly non gyrotropic also before the collision. Then, during the collisions stronger anisotropic/agyrotropic sheets are produced.

Kinetic physics: Non-Maxwellian features



$$\varepsilon(x,y,t)=\frac{1}{n}\sqrt{\int\left[f(\mathbf{x},\mathbf{u},t)-f_M(\mathbf{x},\mathbf{u},t)\right]^2 d^3\mathbf{u}}$$





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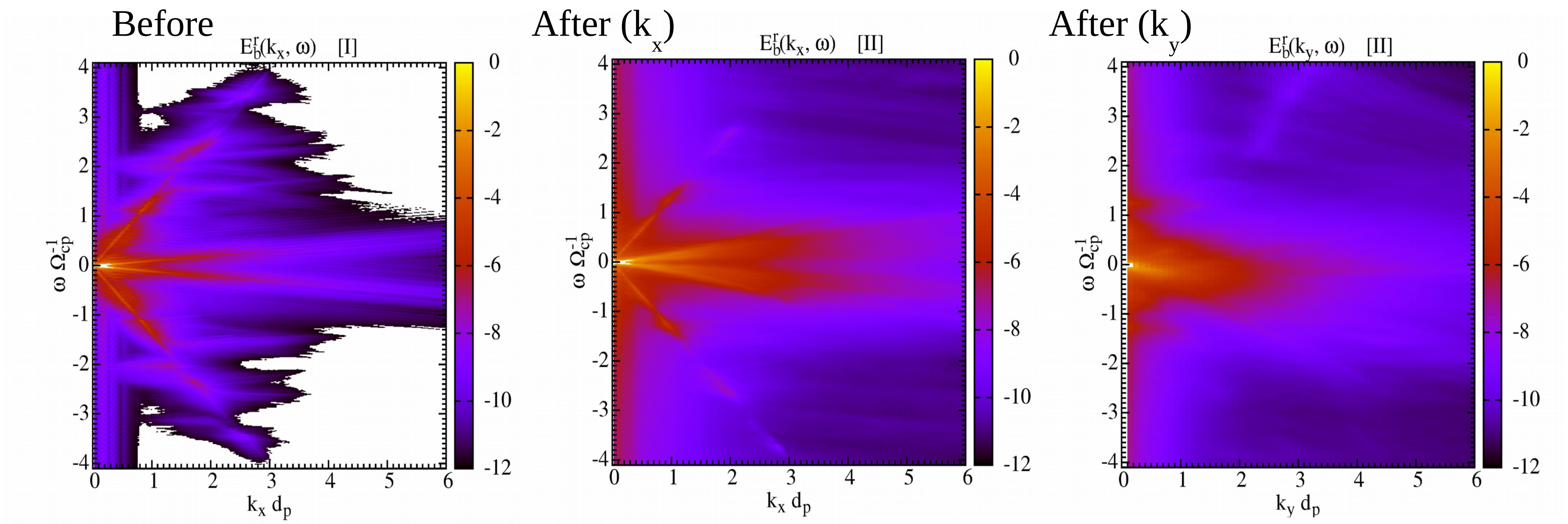
… associated with the production of temperature anisotropy, the VDF displays a beam along the local magnetic field ...

Weakly nonlinear coupling vs turbulence

NO clear dispersion relations can be recovered after the collision...



Before

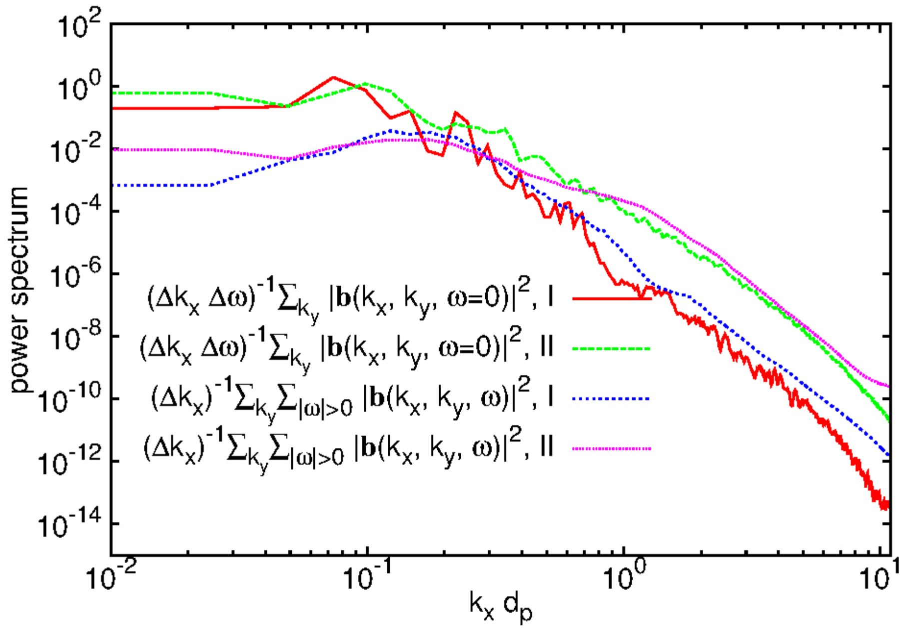
After (k_x)

After (k_y)

Small scales fluctuations both at ω=0 and ω≠0 are generated after the collision:

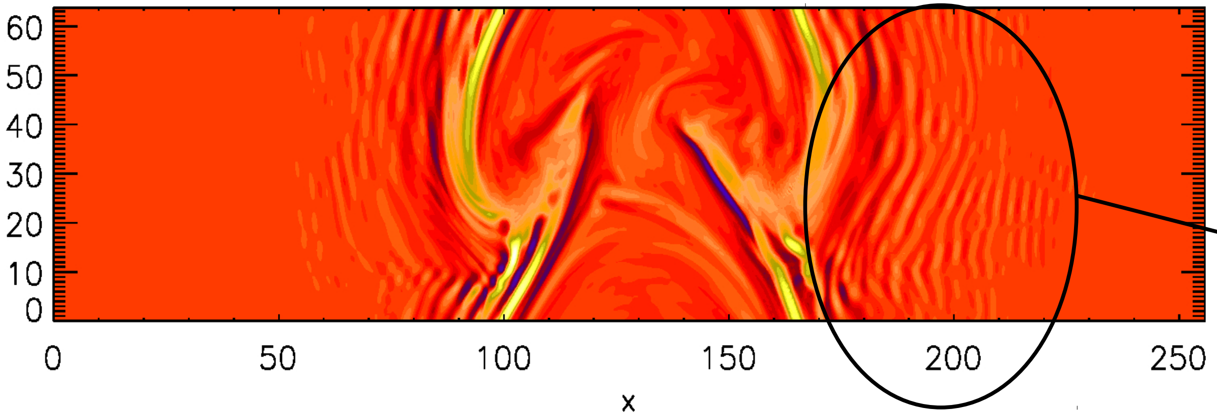
1) ω=0 ⟶ Turbulent, quasi-stationary structures

2) ω≠0 ⟶ Wave-packets tend to produce smaller scales

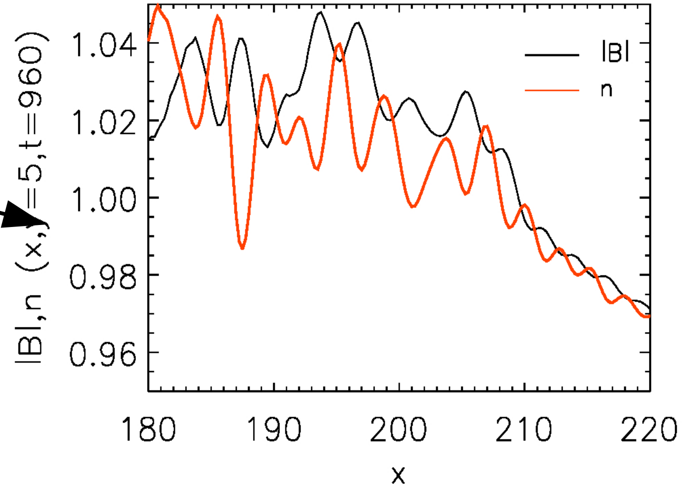


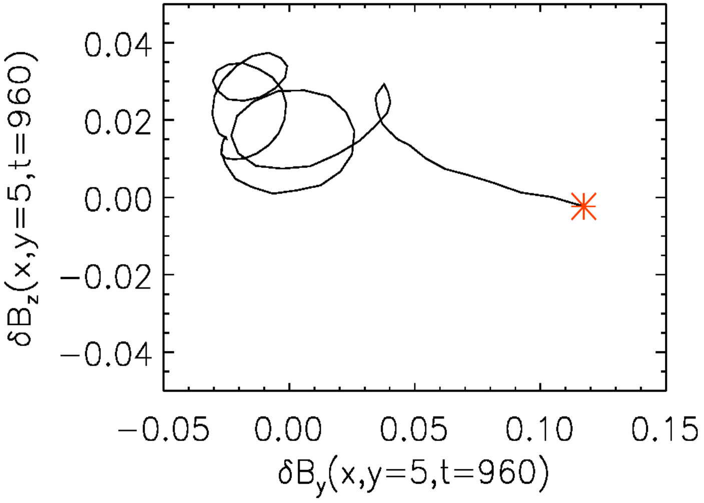
Small scales fluctuations nature

The correlation, polarization and propagation speed of the small-scale ripples which are visible, before the main wave-packet, after the collision are comparable with the ones of the KAW, obtained through a linear two-fluid solver.



[Hollweg, JGR 1999]
[Vasconez, APJ 2015]





However, due to the lack of a clear dispersion relation, weakly nonlinear coupling model may give only a partial interpretation of the phenomena which are occurring in the system.

Conclusions

1) The general scenario described by **Moffatt & Parker is quite well confirmed** in the MHD case:
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2) Other features can be appreciated:
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3) During the evolution, **anisotropic/agyrotropic regions** are produced and a beam is present in the VDF. Moreover, the k-omega analysis suggests that, after the collision, **the dispersion relation shows a significant broadening** and ω=0 fluctuations power increases. Finally, some low-energy fluctuations whose correlations are similar to the KAW ones display after the collision and before the main structure of the wave-packets.

HVM simulations have been run on the **Fermi** parallel machine at **Cineca** (Italy), within the project COLTURBO—HP10CVRU0Q.