

Ultrafast spin  
dynamics in  
ferromagnetic  
films

J.Hurst

Introduction :  
The Vlasov  
equation in  
solid state  
physics

Vlasov  
equation with  
spins

Applications  
to  
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Conclusions  
and future  
investigations

# Ultrafast spin dynamics in ferromagnetic films

Vlasovia 2016

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# Outlines

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- 2 Vlasov equation with spins
- 3 Applications to ferromagnetic films
- 4 Conclusions and future investigations

# Introduction: Motivations

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The Vlasov equation in solid state physics

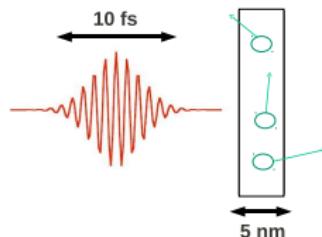
Vlasov equation with spins

Applications to ferromagnetic films

Conclusions and future investigations

- Theoretical aspect :

- Ultrafast ( $\sim 100 \text{ fs} = 10^{-13} \text{ s}$ ) magnetization dynamics at the nanometric scale
- Spin and relativistic effects (Zeeman + Spin-orbit coupling)
- Non linear effects
- Mean field model



- Experimental aspect :

- Ultrafast demagnetization in ferromagnetic films induced by a femtosecond laser pulse : (E. Beaurepaire et al. Phys. Rev. Lett. 76, 4250 1996)

# Introduction: Description of the system

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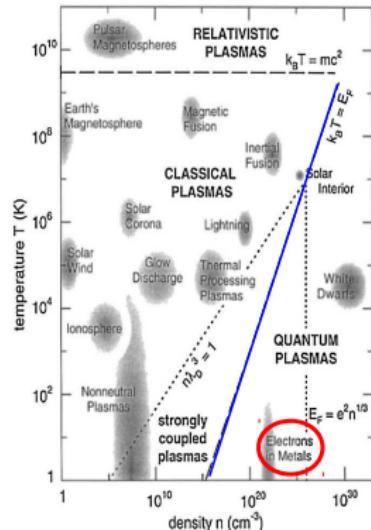
## Thin ferromagnetic films of Nickel : $L = 5 - 20 \text{ nm}$ (1D problem)

Two types of charges: electrons & ions :

	Electrons	Ions
Charge	-e	+2e
Density ( $10^{29} \text{ m}^{-3}$ )	1.8	0.9
Temperature (K)	300	300
mass (a.u)	1	$1.1 * 10^5$
$\omega_p^{-1}(\text{fs})$	0.25	330
Magnetic moment ( $\mu_b$ )	0.066	0.54

Ultrafast dynamics ( $\sim 100\text{fs}$ )  $\rightarrow$  Ions fixed  
( $1\text{fs} = 10^{-15}\text{s}$ )

Electrons  $\leftrightarrow$  Quantum plasmas



Shukla and Eliasson, Rev. Mod. Phys., 2011

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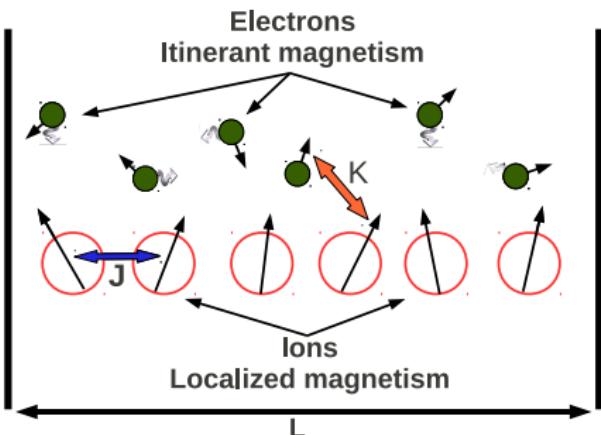
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## Interactions:

- Coulomb
- Magnetic exchange ions-ions ( $J$ )
- Magnetic exchange ions-electrons ( $K$ )

Effective magnetic field  
(100 - 1000 Tesla)



# Introduction: Wigner and Vlasov equations

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Phase-space quantum mechanics (Equivalent to Schrödinger)

Wigner without spins:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} = \frac{ime}{2\pi\hbar^2} \int ds dv' e^{-im(v'-v)s/\hbar} \left[ V_H \left( x + \frac{s}{2}, t \right) - V_H \left( x - \frac{s}{2}, t \right) \right] f(x, v', t)$$

$$\nabla^2 V_H = \frac{e}{\epsilon_0} \left( \int f(x, v, t) dv - n_i \right)$$

Semi-classical limit  $\left( \frac{\hbar}{m\langle v \rangle_e} \ll L \right) \rightarrow$  Vlasov equation :

$$\frac{\partial f}{\partial t} + v \partial_x f + \frac{e}{m} \partial_x V_H \cdot \partial_v f = 0$$

# Wigner function with spin

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Generalisation of the Wigner function :

$$f(\mathbf{r}, \mathbf{v}, t) \rightarrow \mathcal{F}(\mathbf{r}, \mathbf{v}, t) \quad \mathcal{F} = \begin{pmatrix} f^{\uparrow\uparrow} & f^{\uparrow\downarrow} \\ f^{\downarrow\uparrow} & f^{\downarrow\downarrow} \end{pmatrix}$$

Projection onto the Pauli matices:

$$\mathcal{F} = \frac{1}{2}\sigma_0 f_0 + \frac{1}{\hbar}\mathbf{f} \cdot \boldsymbol{\sigma}$$

with

$$f_0 = \text{tr} \{ \mathcal{F} \} = f^{\uparrow\uparrow} + f^{\downarrow\downarrow} \quad \mathbf{f} = \frac{\hbar}{2} \text{tr} (\mathcal{F} \boldsymbol{\sigma})$$

Physical observable :  $i, \alpha = \{x, y, z\}$

$$n(\mathbf{r}, t) = \int f_0(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \quad m_i(\mathbf{r}, t) = \int f_i(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

$$J_i(\mathbf{r}, t) = \int v_i f_0(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \quad J_{i\alpha}^s(\mathbf{r}, t) = \int v_i f_\alpha(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

# Vlasov with spin

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Zeeman + Spin-orbit hamiltonian :

$$\mathcal{H} = \left[ \frac{(P + e\mathbf{A})^2}{2m} + V(\mathbf{r}, t) \right] \sigma_0 + \mu_B \boldsymbol{\sigma} \cdot \mathbf{B}(\mathbf{r}, t)$$
$$+ \frac{\mu_B}{8mc^2} \boldsymbol{\sigma} \cdot [\mathbf{E} \times (\mathbf{p} + e\mathbf{A}) - (\mathbf{p} + e\mathbf{A}) \times \mathbf{E}]$$

Wigner + semi-classical limit

$i = \{x, y, z\}$

$$\frac{\partial f_0}{\partial t} + \mathbf{v} \cdot \nabla f_0 - \frac{e}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_0 + \frac{\mu_B}{2mc^2} (\mathbf{E} \times \nabla)_i f_i$$
$$- \frac{\mu_B}{m} \nabla \left[ \mathbf{B}_i - \left( \frac{\mathbf{v} \times \mathbf{E}}{2c^2} \right)_i \right] \cdot \nabla_{\mathbf{v}} f_i - \frac{\mu_B e}{2m^2 c^2} [\mathbf{E} \times (\mathbf{B} \times \nabla_{\mathbf{v}})]_i f_i = 0$$

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla f_i - \frac{e}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_i + \frac{\mu_B}{2mc^2} (\mathbf{E} \times \nabla)_i f_0$$
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$$- \frac{2\mu_B}{\hbar} \left\{ \left[ \mathbf{B} - \frac{1}{2c^2} (\mathbf{v} \times \mathbf{E}) \right] \times \mathbf{f} \right\}_i = 0.$$

# Maxwell equations

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## Maxwell equations with spin corrections

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} - \frac{\nabla \cdot \mathbf{P}}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \frac{\partial \mathbf{P}}{\partial t} + \mu_0 \nabla \times \mathbf{M}$$

Source terms depend on the spin (Semi-relativistic expansion)

$$\rho = -e \int f_0 d\mathbf{v}, \quad \mathbf{j} = -e \left[ \int \mathbf{v} f_0 d\mathbf{v} + \frac{\mathbf{E} \times \mathbf{M}}{2mc^2} \right]$$

$$\mathbf{M} = -\mu_B \int \mathbf{f} d\mathbf{v}, \quad \mathbf{P} = -\frac{\mu_B}{2c^2} \int \mathbf{v} \times \mathbf{f} d\mathbf{v}$$

A Dixit, Y. Hinschberger, J. Zamanian, G. Manfredi, et P.-A Hervieux, Phys. Rev. A 88, 032117 (2013).

# Minimal self consistent model

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1d-1v distribution function  $(x, v_x) \rightarrow (\text{no spin-orbite})$

$$\frac{\partial f_0}{\partial t} + v \frac{\partial_x f_0}{m} + \frac{e}{m} (\partial_x \phi) (\partial_v f_0) - \frac{\mu_b}{m} \partial_x \left( B_i - \frac{Kn_i S_0}{2\mu_b} S_i \right) \partial_v f_i = 0$$

$$\frac{\partial f_i}{\partial t} + v \frac{\partial_x f_i}{m} + \frac{e}{m} (\partial_x \phi) (\partial_v f_i) - \frac{\mu_b}{m} \partial_x \left( B_i - \frac{Kn_i S_0}{2\mu_b} S_i \right) \partial_v f_0$$

$$- \frac{KN_0 S_0}{\hbar} \{ \mathbf{f} \times \mathbf{S} \}_i - \frac{2\mu_B}{\hbar} \{ \mathbf{B} \times \mathbf{f} \}_i = 0$$

+ Electrostatic limit of Maxwell equations :  $i = \{x, y, z\}$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\epsilon_0} (n(x) - n_{ions}) \quad \frac{\partial B_i}{\partial x} = -\mu_B \mu_0 \frac{\partial}{\partial x} m_i(x) \quad n = \int f_0 d\nu \quad m_i = \int f_i d\nu$$

Landau Lifchitz Gilbert (LLG) equation for the magnetic moment of ions :

$$\frac{\partial \mathbf{S}(x)}{\partial t} = \frac{a^2 J S_0}{\hbar} \left[ \mathbf{S}(x) \times \frac{\partial^2}{\partial x^2} \mathbf{S}(x) \right] + \frac{K}{2\hbar} \left[ \mathbf{S} \times \int \mathbf{f} d\nu \right] - \gamma \mathbf{S} \times \mathbf{B}_{ext}$$

# Ground state : Thin film of Nickel

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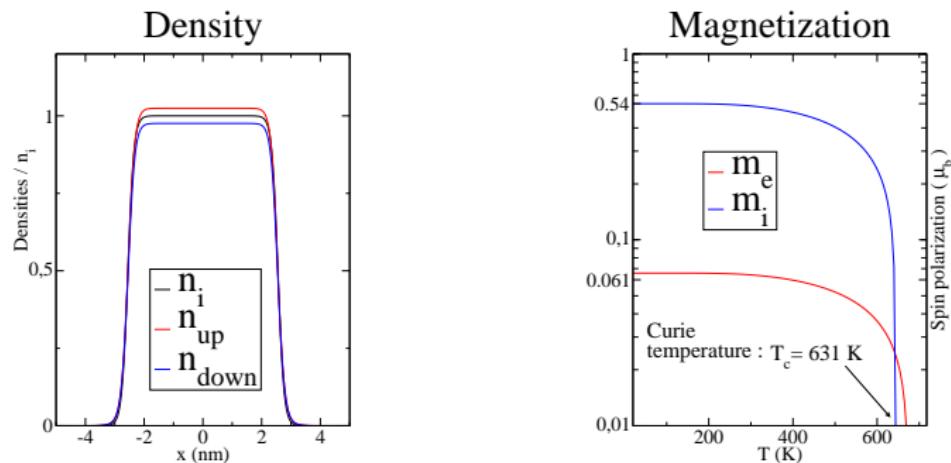
Conclusions and future investigations

Collinear magnetism + Pauli exclusion principle :

$$f_0^{\text{ground}} = \mathcal{F}_D^+ + \mathcal{F}_D^-, \quad f_z^{\text{ground}} = \mathcal{F}_D^+ - \mathcal{F}_D^-$$

$$\mathcal{F}_D^\pm = \frac{2\pi k_b T}{m} \left( \frac{m}{2\pi\hbar} \right) \ln \left[ 1 + \exp \left( -\frac{1}{k_b T} \left( \frac{m}{2} v^2 - e\phi \pm \mu_B B_z - \mu(T) \right) \right) \right]$$

Ions :  $\langle \mathbf{S} \rangle = S \mathcal{B}_S (\beta S h_{\text{eff}}), \quad h_{\text{eff}} = \frac{\kappa}{2} m_e(\mathbf{r}) + J N_v \langle \mathbf{S} \rangle + g \mu_B \mathbf{H}$



# Linear regime : Film of Nickel ( $\sim 5\text{nm}$ )

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Laser Excitation :  $E(t) = E_0 \sin(\omega t) e^{-[(t-t_0)/\Delta t]^2}$   $E_0 \sim 10^8 \text{V/m}$

Two modes : Plasmon  $\omega_p = \sqrt{e^2 n / m \epsilon_0}$  & Ballistic  $\omega_b = (2L/v_F)^{-1}$

Observable :  $D_{electric} = \int x f_0 dx dv$  ;  $D_{magnetic} = \int x f_z dx dv$

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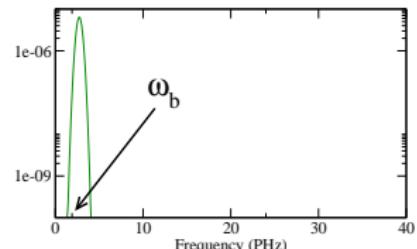
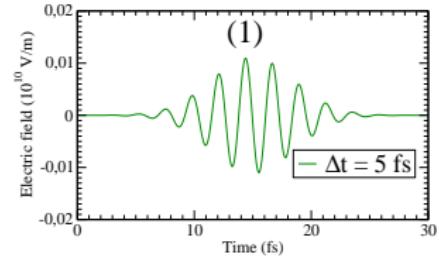
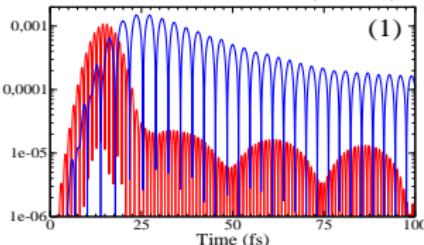
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$\Delta t = 5 \text{ fs}$  ;  $\omega = 2.73 \text{ PHz}$  (800 nm)



# Linear regime : Film of Nickel ( $\sim 5\text{nm}$ )

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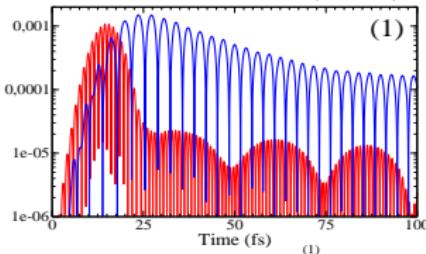
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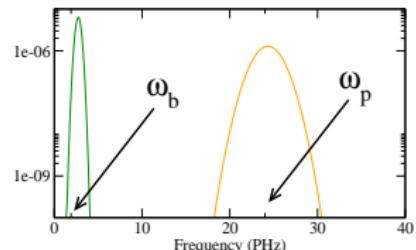
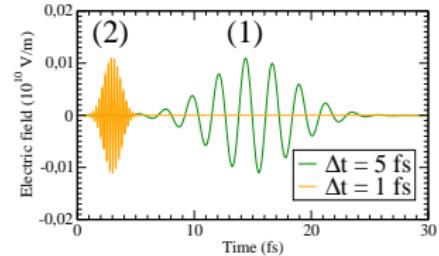
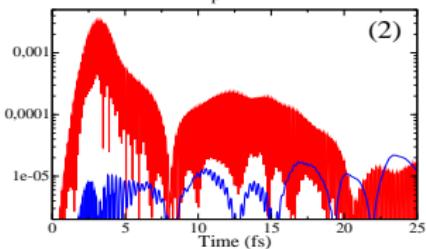
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$\Delta t = 5 \text{ fs}$  ;  $\omega = 2.73 \text{ PHz}$  (800 nm)



$\Delta t = 1 \text{ fs}$  ;  $\omega = \omega_p = 24.4 \text{ PHz}$  (77 nm)



# Initial distribution function : Water bag model

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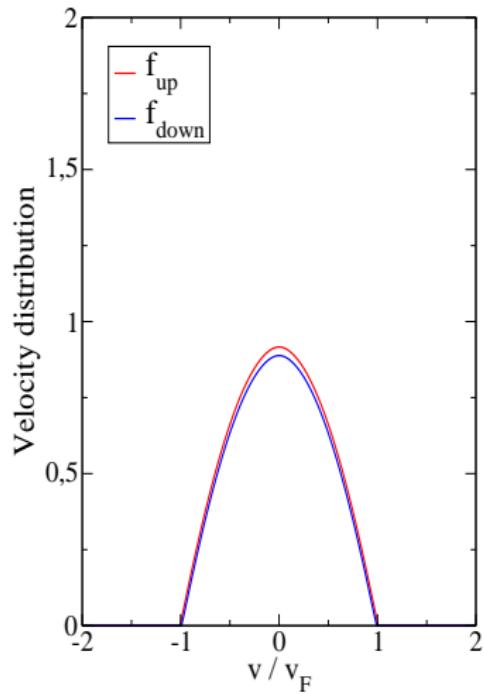
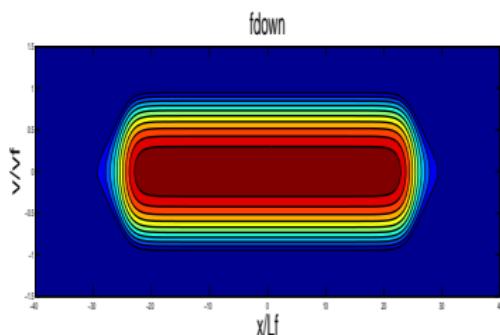
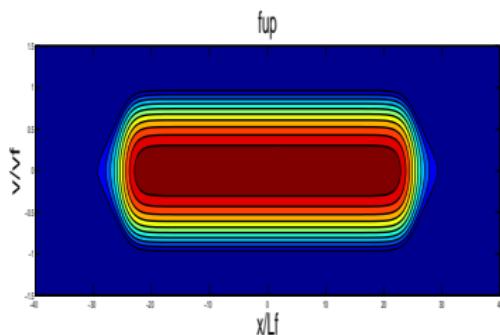
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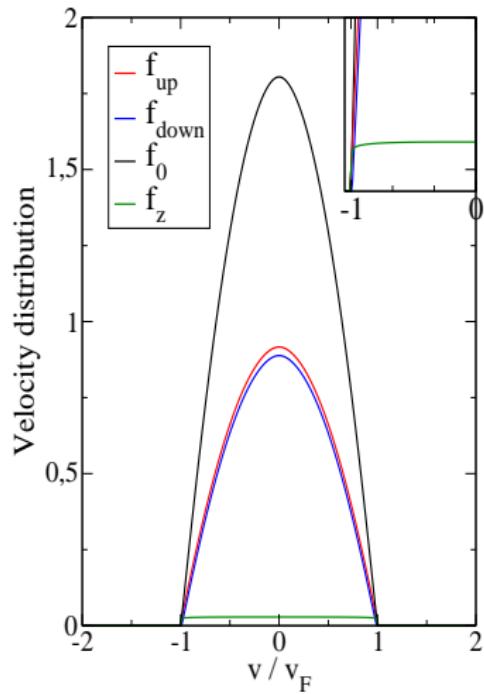
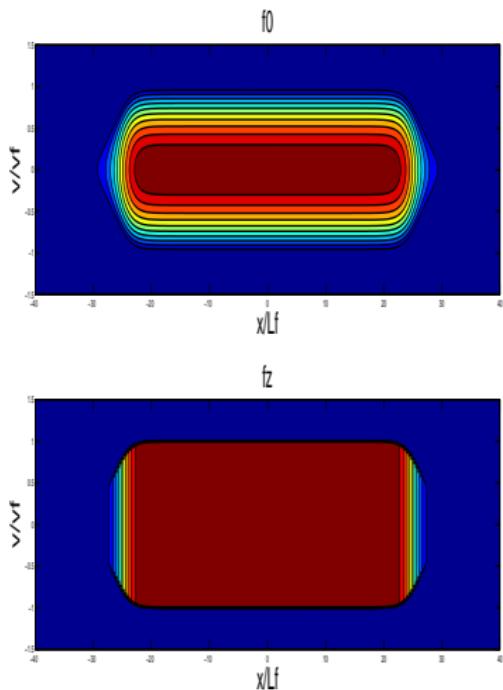
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# Non linear regime

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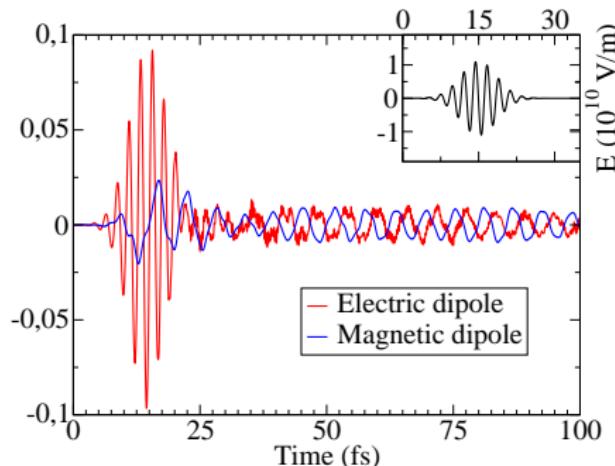
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$$E(t) = E_0 \sin(\omega t) \exp\left(-[(t - t_0)/\Delta t]^2\right)$$

$$E_0 \sim 1.1 \cdot 10^{10} \text{ V/m} ; \Delta t = 5 \text{ fs} ; \omega = 2.73 \text{ PHz (800 nm)}$$

Plasmon mode → Non linear effects

The magnetic dipole remains



# Conclusions and future investigations

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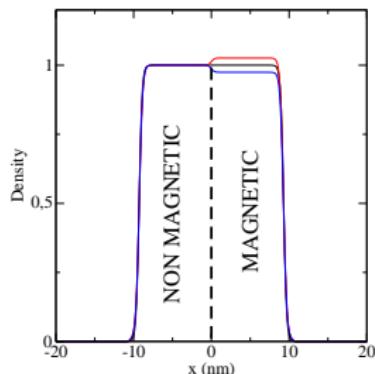
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## Conclusions :

- We built a self consistent model to study ultrafast magnetization dynamics in thin ferromagnetic films
- We show the existence of an oscillating magnetic dipole at ultra high frequency (PHz) **NEW**

## Futur investigations:

- Non collinear dynamics
- Non magnetic/magnetic bilayers
- Extension to 4D (Spin orbit)



# Acknowledgement

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