# SELF-CONSISTENT PLASMA WAKE FIELD DYNAMICS OF A CHARGED-PARTICLE BEAM: VLASOV vs QUANTUM-LIKE APPROACHES

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# ❑ FORMULATION OF THE PHYSICAL PROBLEM

## SELF-CONSISTENT PLASMA WAKE FIELD (PWF) EXCITATION

**LONG REL. CHARGED-PARTICLE BEAM**
traveling along z-axis

**COLD PLASMA**
(*active medium*)

**PWF excitation**

**PLASMA ELECTRIC FIELD WITH A RADIAL PROFILE**
(*PWF-driven plasma wake*)

$$\mathbf{F}_w = -\nabla_\perp U_w$$

- PARAXIAL BEAM
- NON-LAMINAR BEAM: finite temperature/emittance
- LONG BEAM LIMIT: $\sigma_z >> \lambda_p$
- PRESENCE OF A STRONG EXTERNAL UNIFORM MAGNETIC FIELD: $B_0$ // $z$

➡ *Self-consistent beam-plasma interaction or beam self-modulation*

# ❑ BASIC ASSUMPTIONS ON *PLASMA + CHARGED PARTICLE BEAM*

- Plasma
  - Collisionless
  - Magnetized: strong constant and uniform external magnetic field ($\mathbf{B}_0 = B_0\, \mathbf{e}_z$)
  - Overdense regime: $n_0 >> n_b$
    $n_0$ = unperturbed plasma density
    $n_b$ = unperturbed beam density
  - The ions are supposed infinitely massive and constitute a background of positive charge with density $n_0$
- Electron/positron Beam
  - Relativistic, travelling along the magnetic field
  - The beam length is much greater than the plasma wavelength (*long beam limit*)

***The entire beam experiences the effects of the plasma wake fields (PWF) that itself has produced (self interaction)***

# ❑ PLASMA MODEL: THE LORENTZ-MAXWELL SYSTEM

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{e}{m_0}\mathbf{E} - \frac{e}{m_0 c}\mathbf{u}\times\mathbf{B}$$

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}) = 0$$

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\left[q\rho_b\left(\beta c\hat{z} + \mathbf{u}_{b\perp}\right) - en\mathbf{u}\right] + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla\cdot\mathbf{E} = 4\pi e(n_0 - n) + 4\pi q\rho_b$$

- We express $\mathbf{E}$ and $\mathbf{B}$ in terms of the 4-potential $(\mathbf{A},\phi)$ and linearize the *system "plasma + beam" by introducing small perturbations*

  [P. Chen, Part. Accel. **20**, 171 (1987), P. Chen, J. J. Su, T. Katsouleas, S. Wilks, and J. M. Dawson, IEEE Transactions on Plasma Science **15**, 218 (1987)]

- We transform all the system of equations to the beam co-moving frame $\xi = z\text{-}\beta ct \simeq z\text{-}ct$ $(\beta \simeq 1)$
- We split $\mathbf{p}$ and $\mathbf{A}$ into the longitudinal and transverse components, viz., $\mathbf{p} = \mathbf{z}\, p_z + \mathbf{p}_\perp$, $\mathbf{A} = \mathbf{z}\, \mathrm{A}_z + \mathbf{A}_\perp$

# POISSON-TYPE EQUATION

$$\boxed{\left[\left(\frac{\partial^2}{\partial\xi^2}+k_{uh}^2\right)\left(\nabla_\perp^2-k_p^2\right)+k_p^2k_B^2\right]\Omega=-k_p^4\frac{qm_{e0}c^2}{e^2}\frac{\rho_b}{n_0}}$$

$k_{uh}=\left(k_p^2+k_B^2\right)^{1/2}$ $\quad k_p=\left(4\pi e^2n_0/m_{e0}c^2\right)^{1/2}\equiv\omega_p/c$ $\quad k_B=\omega_B/c\equiv qB_0/m_{e0}c^2$

$\Omega\left(\mathbf{r}_\perp,\xi\right)=A_{1z}\left(\mathbf{r}_\perp,\xi\right)\beta-\phi_1\left(\mathbf{r}_\perp,\xi\right)$ *wake potential*

- **Long beam**, i.e., $\sigma_z k_{uh}\gg 1$

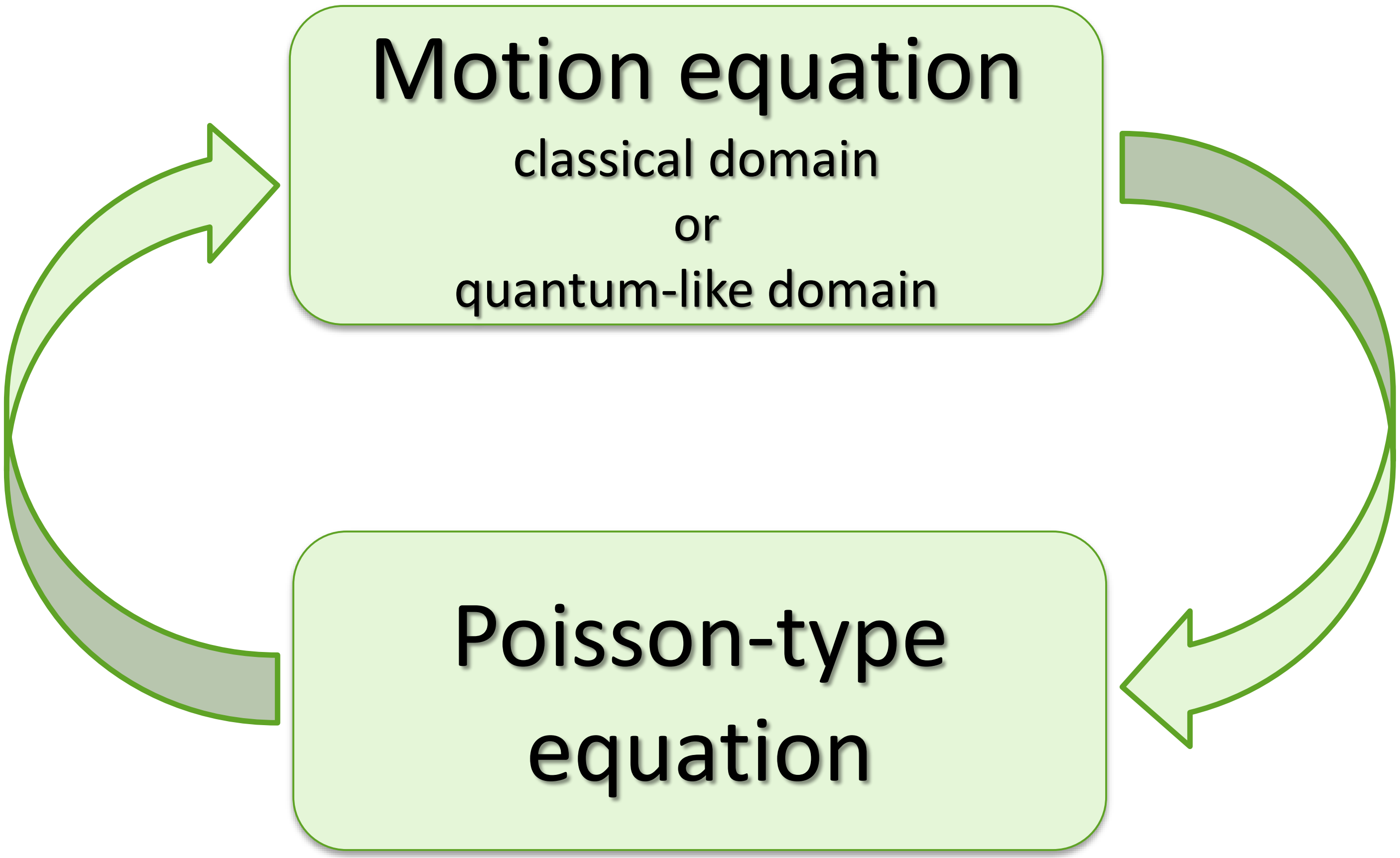
$$\boxed{\left(\nabla_\perp^2-k_s^2\right)U_w=k_s^2\frac{\rho_b}{\gamma_0 n_0}}$$

$k_s=k_p^2/k_{uh}$ $\quad q=\pm e$

$U_w\left(\mathbf{r}_\perp,\xi\right)=-\frac{q\Omega\left(\mathbf{r}_\perp,\xi\right)}{m_{e0}\gamma_0c^2}$

*wake potential energy*

$$U_w=U_w\left[\rho_b\right]$$




$$\mathbf{F}_w = -\nabla_\perp U_w$$

# ❑ BEAM DYNAMICS: SINGLE BEAM-PARTICLE HAMILTONIAN

- The relativistic single-particle Hamiltonian associated with the perturbed transverse dynamics of the beam including ***external magnetic field + interaction with plasma***, can be expressed in terms of the four-potential $(\boldsymbol{A},\phi)$:

$$\boxed{H = c\left[(\mathbf{p} - \frac{q}{c}\mathbf{A})^2 + m_0^2c^2\right]^{\frac{1}{2}} + q\phi}$$

$$\Downarrow$$

*Effective single-particle Hamiltonian*

$$\boxed{\mathcal{H} = \frac{1}{2}\mathcal{P}_\perp^2 + \frac{1}{2}k_c\hat{z}\cdot(\mathbf{r}_\perp \times \mathcal{P}_\perp) + U_w(\mathbf{r}_\perp,\xi) + \frac{1}{2}Kr_\perp^2}$$

$$\mathcal{H} = \Delta H/H_0 = (H - H_0)/H_0 \qquad \mathcal{P}_\perp = \mathbf{p}_\perp/m_0\gamma_0 c \quad k_c = -qB_0/m_0\gamma_0 c^2$$

$$H_0 = c(p_0^2 + m_0^2c^2)^{1/2} = m_0\gamma_0 c^2 \qquad K = (k_c/2)^2 = \omega_B^2/4\gamma_0^2c^2$$

## Classical domain: *Vlasov – Poisson-type system of equations*

$$\frac{\partial f}{\partial \xi} + \left[\mathbf{p}_\perp + \frac{1}{2}k_c(\hat{z} \times \mathbf{r}_\perp)\right] \cdot \frac{\partial f}{\partial r_\perp} - \left[K\mathbf{r}_\perp + \frac{\partial U_w}{\partial \mathbf{r}_\perp} - \frac{1}{2}k_c(\hat{z} \times \mathbf{p}_\perp)\right] \cdot \frac{\partial f}{\partial \mathbf{p}_\perp} = 0$$

$$\nabla_\perp^2 U_w - k_s^2 U_w = \frac{k_s^2}{n_0\gamma_0}\rho_b$$

$$\rho_b(\mathbf{r}_\perp, \xi) = \frac{N}{\sigma_z}\int f(\mathbf{r}_\perp, \mathbf{p}_\perp, \xi)\ d^2\mathbf{p}_\perp$$

*N = number of beam particles*

$$\Rightarrow \quad U_w = U_w\left[f\right]$$

# ❑ Quantum-like domain: *Thermal Wave Model* (TWM)

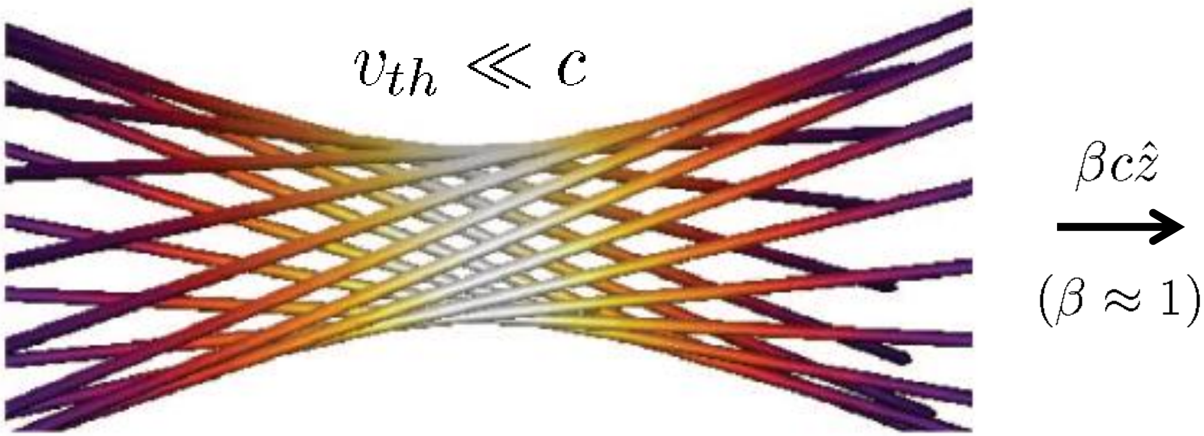
- It provides an *effective description* of the transverse dynamics of a relativistic charged particle beam, of ***transverse emittance*** $\varepsilon$, in terms of a complex wave function $\Psi(\mathbf{r}_\perp, \xi)$, called ***beam wave function*** (BWF), whose squared modulus is proportional to the beam density, i.e., $\rho_\mathrm{b}(\mathbf{r}_\perp, \xi) \propto |\Psi(\mathbf{r}_\perp, \xi)|^2$
- According to TWM the following *Schrödinger-like equation* for BWF can be assumed:

$$\boxed{i\epsilon \frac{\partial \Psi}{\partial \xi} = \mathcal{H}\left(\mathbf{r}_\perp, -i\epsilon\nabla_\perp, \xi\right)\Psi}$$

[R. Fedele and G. Miele, *Il Nuovo Cimento D* **13,** 1527 (1991)]

## ➢ Mixing among the electron rays: in vacuo and absence of forces

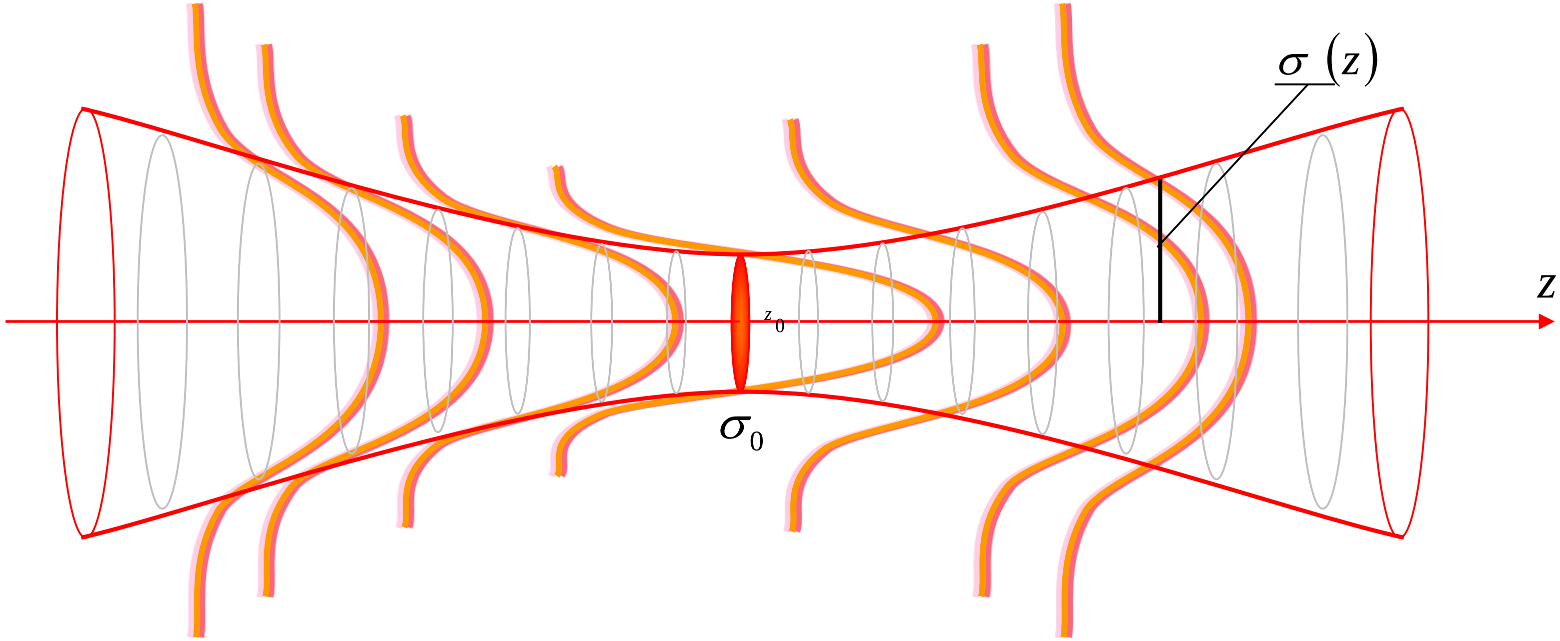
The dispersion among the particle trajectories (electron rays) is due to the *thermal agitation* (*thermal spreading*). If the thermal velocity is much smaller than the speed of light, the beam is paraxial and the mixing of the electron rays provides a picture fully similar to one of the light ray mixing due to the paraxial diffraction.




$$\boxed{i\epsilon\frac{\partial\Psi}{\partial\xi} = -\frac{\epsilon^2}{2}\nabla_\perp^2\Psi}$$

$$\sigma\sigma_p \geq \epsilon$$

- **Qualitative envelope evolution of a cilindrically-symmetric Gaussian beam propagating in vacuo**




$$\sigma(z) = \sqrt{\sigma_0^2 + \frac{\epsilon^2}{\sigma_0^2}(z - z_0)^2}$$

# Quantum-like paraxial diffraction

$$i\varepsilon\frac{\partial\Psi}{\partial\tau}=-\frac{\varepsilon^2}{2}\nabla_{\perp}{}^2\Psi+U(x,y,\tau)\Psi$$

$$\sigma\sigma_p\geq\epsilon$$

If $\langle q_i\rangle=\langle p_i\rangle=0$

$$\epsilon_i(\tau)=2\left[\left\langle q_i^2\right\rangle\left\langle p_i^2\right\rangle-\langle q_i p_i\rangle^2\right]^{1/2}$$

It can be proven that, if $A_i(\tau)$ is the instantaneous area occupied by the beam in the 2D subspace (qi,pi), the following identification holds:

$$\epsilon_i(\tau)=\frac{A_i(\tau)}{\pi}\geq 0$$

## ❑ **Quantum-like domain: *Schrödinger – Poisson-type system of equations*** or ***Zakharov-type system of equations***

$$i\epsilon\frac{\partial\Psi}{\partial\xi} = -\frac{\epsilon^2}{2}\nabla_\perp^2\Psi - \frac{i\epsilon k_c}{2}\hat{z}\cdot(\mathbf{r}_\perp\times\nabla_\perp)\Psi + U_w(\mathbf{r}_\perp,\xi)\Psi + \frac{1}{2}Kr_\perp^2\Psi$$

$$\nabla_\perp^2 U_w - k_s^2 U_w = \frac{k_s^2}{n_0\gamma_0}\rho_b$$

$$\rho_b\left(\mathbf{r}_\perp,\xi\right) = \frac{N}{\sigma_z}\left|\Psi\left(\mathbf{r}_\perp,\xi\right)\right|^2$$

*N = number of beam particles*

$$\Rightarrow \quad \mathrm{U}_w = U_w\left[\left|\Psi\right|^2\right]$$

# Classical vs quantum-like

$$\nabla_\perp^2 U_w - k_s^2 U_w = \frac{k_s^2}{n_0\gamma_0}\rho_b \qquad H(\mathbf{r}_\perp,\mathbf{p}_\perp,\tau) = \frac{1}{2}p_\perp^2 + \frac{1}{2}k_c\,\hat{z}\cdot(\mathbf{r}_\perp\times\mathbf{p}_\perp) + \frac{1}{2}Kr_\perp^2 + U_w(\mathbf{r}_\perp,\tau)$$

$$H(\mathbf{r}_\perp,\mathbf{p}_\perp,\tau) \longrightarrow \hat{H} = H\left(\mathbf{r}_\perp, -i\epsilon\frac{\partial}{\partial\mathbf{r}_\perp},\xi\right)$$

| CLASSICAL DOMAIN | QUANTUM-LIKE DOMAIN |
|---|---|
| $\frac{\partial f}{\partial\tau} + \frac{\partial H}{\partial\mathbf{p}_\perp}\cdot\frac{\partial f}{\partial\mathbf{r}_\perp} - \frac{\partial H}{\partial\mathbf{r}_\perp}\cdot\frac{\partial f}{\partial\mathbf{p}_\perp} = 0$ | $i\epsilon\frac{\partial\Psi}{\partial\tau} = \hat{H}\Psi$ |
| $\rho_b(\mathbf{r}_\perp,\tau) = \frac{N}{\sigma_z}\int f(\mathbf{r}_\perp,\mathbf{p}_\perp,\tau)d^2r_\perp,d^2p_\perp$ | $\rho_b(\mathbf{r}_\perp,\tau) = \frac{N}{\sigma_z}\left\|\Psi(\mathbf{r}_\perp,\tau)\right\|^2$ |
| $\int f(\mathbf{r}_\perp,\mathbf{p}_\perp,\tau)d^2r_\perp,d^2p_\perp = 1$ | $\int \left\|\Psi(\mathbf{r}_\perp,\tau)\right\|^2 d^2r_\perp = 1$ |
| $\langle F\rangle = \int F(\mathbf{r}_\perp,\mathbf{p}_\perp,\tau)f(\mathbf{r}_\perp,\mathbf{p}_\perp,\tau)d^2r_\perp\, d^2p_\perp$ | $\langle\hat{F}\rangle = \int\Psi^*\hat{F}\Psi\, d^2r_\perp$ |

## Wigner quasidistribution and von Neumann equation

$$W\left(\mathbf{r}_\perp, \mathbf{p}_\perp, \tau\right) = \frac{1}{(2\pi\epsilon)^2} \int \Psi^* \left(\mathbf{r}_\perp + \frac{\mathbf{y}}{2}, \tau\right) \Psi \left(\mathbf{r}_\perp - \frac{\mathbf{y}}{2}, \tau\right) \exp\left(\frac{i}{\epsilon} \mathbf{p}_\perp \cdot \mathbf{y}\right) d^2 y$$

$$H\left(\mathbf{r}_\perp, \tau\right) = \frac{p_\perp^2}{2} + U\left(\mathbf{r}_\perp, \tau\right)$$

$$\frac{\partial W}{\partial \tau} + \mathbf{p}_\perp \cdot \frac{\partial W}{\partial \mathbf{r}_\perp} + \frac{i}{\epsilon}\left[U\left(\mathbf{r}_\perp + \frac{i\epsilon}{2}\frac{\partial}{\partial \mathbf{p}_\perp}\right) - U\left(\mathbf{r}_\perp - \frac{i\epsilon}{2}\frac{\partial}{\partial \mathbf{p}_\perp}\right)\right] W = 0$$

$$\langle \hat{F} \rangle = \int F\left(\mathbf{r}_\perp, \mathbf{p}_\perp, \tau\right) W\left(\mathbf{r}_\perp, \mathbf{p}_\perp, \tau\right) d^2 r_\perp d^2 p_\perp$$

Due to the Q.L. uncertainty relation, W can become negative, which corresponds to a loss of information in phase space cells of the order of the emittance

## Virial description for both classical and quantum-like domains

$$\sigma_\perp(\tau) = \langle r_\perp^2 \rangle^{1/2} \quad \sigma_{p_\perp}(\tau) = \langle p_\perp^2 \rangle^{1/2} \quad \mathcal{E}(\tau) = \langle H \rangle$$

$$\Rightarrow \frac{d\sigma_\perp^2}{d\tau} = 2\left\langle \mathbf{r}_\perp \cdot \mathbf{p}_\perp \right\rangle \quad \Rightarrow \sigma_\perp \frac{d\sigma_\perp}{d\tau} = \left\langle \mathbf{r}_\perp \cdot \mathbf{p}_\perp \right\rangle$$

$$\mathcal{L}_z = \langle L_z \rangle, L_z = \hat{z} \cdot (\mathbf{r}_\perp \times \mathbf{p}_\perp)$$

$$U(\mathbf{r}_\perp, \mathbf{p}_\perp, \tau) = \frac{1}{2}k_c\, \hat{z} \cdot (\mathbf{r}_\perp \times \mathbf{p}_\perp) + \frac{1}{2}K r_\perp^2 + U_w(\mathbf{r}_\perp, \tau)$$

$$\Rightarrow \boxed{\begin{gathered} \frac{d^2\sigma_\perp^2}{d\tau^2} = 4(\mathcal{E} - \langle U \rangle) - 2\langle \mathbf{r}_\perp \cdot \nabla_{r_\perp} U \rangle + k_c \mathcal{L}_z \\ \frac{d\mathcal{E}}{d\tau} = \left\langle \frac{\partial U}{\partial \tau} \right\rangle \end{gathered}}$$

## Constants of motion and envelope description

$$C = \frac{1}{2}\sigma_{p\perp}^2 + \frac{1}{2}K\sigma_\perp^2 - \frac{1}{2k_s^2\lambda_0}\int\left(|\nabla_\perp U_w|^2 + k_s^2 U_w^2\right) d^2r_\perp + \frac{1}{2}k_c\mathcal{L}_z$$

$$\frac{d^2\sigma_\perp^2}{d\tau^2} + 4K\sigma_\perp^2 = 4C + \frac{2}{k_s^2\lambda_0}\int |\nabla_\perp U_w|^2\, d^2r_\perp - 2k_c\mathcal{L}_z$$

$$\lambda_0 = N/n_0\gamma_0\sigma_z$$

- Cylindrical symmetry: $\mathcal{A} = C - \frac{1}{2}k_c\mathcal{L}_z$ = *new constant*

$$\boxed{\frac{d^2\sigma_\perp^2}{d\tau^2} + 4K\sigma_\perp^2 = 4\mathcal{A} + \frac{2}{k_s^2\lambda_0}\int |\nabla_\perp U_w|^2\, d^2r_\perp}$$

# Self-modulated beam envelope dynamics

R. Fedele, T. Akhter, D. Jovanovic, S. De Nicola and A. Mannan, Eur. Phys. J. D (2014) 68: 210

$$\frac{d^2\sigma_\perp^2}{d\tau^2} + 4K\sigma_\perp^2 = 4\mathcal{A} + \frac{2}{k_s^2\lambda_0}\int |\nabla_\perp U_w|^2 \, d^2 r_\perp$$

$$\nabla_\perp^2 U_w - k_s^2 U_w = \frac{k_s^2}{n_0\gamma_0}\rho_b$$

**+** Sagdeev potential method

- $k_s\sigma_\perp \gg 1$

**PURELY LOCAL REGIME**

- **Criteria for collapse, betatron oscillations and self-equilibrium established**
- **Concept of Gaussian beam equivalent introduced**

$\mathbf{B}_0 \neq 0$

$$\frac{d^2\sigma_\perp^2}{d\tau^2} + 4K\sigma_\perp^2 = 4\mathcal{A} \Rightarrow \frac{d^2\tilde{\sigma}}{d\tilde{\tau}^2} + \tilde{K}\tilde{\sigma} - \frac{A_0}{\tilde{\sigma}^3} = 0$$

*analog of the beam emittance*

$$A_0 = 2\tilde{\sigma}^2\left(\mathcal{A} - \frac{1}{2}\tilde{\sigma}'^2 - \frac{1}{2}\tilde{K}\tilde{\sigma}^2\right)$$

# Self-modulated beam envelope dynamics

R. Fedele, A. Mannan, S. De Nicola, D. Jovanovic and T. Akhter, Eur. Phys. J. D (2014) 68: 271;
R. Fedele, D.Jovanović, F. Tanjia, S. De Nicola, Nucl. Instr. Meth. Phys. Res.A 740 (2014) 180–185

- $k_s \sigma_\perp \gg 1$
  - **STRONGLY NONLOCAL REGIME**
  - **Criteria for collapse, self-defocusing/self-focusing and self-equilibrium established**
  - **Concept of Gaussian beam equivalent introduced**

$\mathbf{B}_0 = 0$

$$\frac{d^2\sigma_\perp^2}{d\tau^2} = 4\mathcal{A} \quad \Rightarrow \quad \frac{d^2\tilde{\sigma}}{d\tilde{\tau}^2} - \frac{A_0}{\tilde{\sigma}^3} = 0$$

*analog of the beam emittance* $\quad A_0 = 2\tilde{\sigma}^2\left(\mathcal{A} - \frac{1}{2}\tilde{\sigma}'^2\right)$

# ❑ Self-modulated beam envelope dynamics

T. Akhter, R. Fedele, S. De Nicola, F. Tanjia, D. Jovanovic and A. Mannan, *Self-modulated dynamics of a relativistic charged particle beam in plasma wake field excitation*, Nucl. Instr. Meth. A, to appear (2016)
D. Jovanovic, R. Fedele, F. Tanjia, S. De Nicola, and M. Belic, EPL, 100 (2012) 55002
R. Fedele, F. Tanjia, S. De Nicola, D. Jovanović, and P. K. Shukla, Phys. Plasma 19, 102106 (2012)

- $k_s \sigma_\perp \ll 1$
  - **Criteria for betatron oscillations and self-equilibrium established**
  - **Collapse prevented**
  - **Stabilizing role of the magnetic field**
- **General case, including** $k_s \sigma_\perp \sim 1$
  - **Self-modulation instability in plasma wake field accelerator predicted**
  - **Stabilizing role of the magnetic field**

PHYSICAL REVIEW E VOLUME 58, NUMBER 1 JULY 1998

# Quantumlike corrections and semiclassical description of charged-particle beam transport

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(Received 6 October 1997)

It is shown that the standard classical picture of charged-particle beam transport in paraxial approximation may be conveniently replaced by a Wigner-like picture in a *semiclassical approximation*. In this *effective* description, the classical phase-space equation for electronic rays is replaced by a *von Neumann–like equation*, where the transverse emittance plays the role of $\hbar$. Relevant remarks concerning the quantumlike corrections for an arbitrary potential in comparison with the standard classical description of the beam transport are given. [S1063-651X(98)07506-0]

$$\hat{\mathcal{L}} \equiv \frac{\partial}{\partial z} + p\frac{\partial}{\partial x} - \left(\frac{\partial U}{\partial x}\right)\frac{\partial}{\partial p}$$

| CLASSICAL DOMAIN | QUANTUM-LIKE DOMAIN |
|---|---|
| $\hat{\mathcal{L}}\rho_w = 0$ | $\hat{\mathcal{L}}\rho_w = \sum_{k=1}^{\infty}\frac{(-1)^k}{(2k+1)!}\left(\frac{\epsilon}{2}\right)^{2k}\frac{\partial^{2k+1}U}{\partial x^{2k+1}}\frac{\partial^{2k+1}\rho_w}{\partial p^{2k+1}}$ |

**The fluid theories generated in classical and quantum-like domains, respectively, are indistinguishable up to the third-order moments**

PHYSICAL REVIEW E VOLUME 60, NUMBER 5 NOVEMBER 1999

# Role of semiclassical description in the quantumlike theory of light rays

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(Received 29 December 1998; revised manuscript received 1 July 1999)

An alternative procedure to the one by Gloge and Marcuse [J. Opt. Soc. Am. **59**, 1629 (1969)] for performing the transition from geometrical optics to wave optics in the paraxial approximation is presented. This is done by employing a recent "deformation" method used to give a quantumlike phase-space description of charged-particle-beam transport in the semiclassical approximation. By taking into account the uncertainty relation (diffraction limit) that holds between the transverse-beam-spot size and the rms of the light-ray slopes, the classical phase-space equation for light rays is deformed into a von Neumann–like equation that governs the phase-space description of the beam transport in the semiclassical approximation. Here, $\hbar$ and the time are replaced by the inverse of the wave number, $\lambda\!\!\!^{-}$, and the propagation coordinate, respectively. In this framework, the corresponding Wigner-like picture is given and the quantumlike corrections for an arbitrary refractive index are considered. In particular, it is shown that the paraxial-radiation-beam transport can also be described in terms of a fluid motion equation, where the pressure term is replaced by a quantumlike potential in the semiclassical approximation that accounts for the diffraction of the beam. Finally, a comparison of this fluid model with Madelung's fluid model is made, and the classical-like picture given by the tomographic approach to radiation beams is advanced as a future perspective. [S1063-651X(99)18110-8]

## Quantum-like corrections and role of the dispersion (sort of deformation method)

$$\eta \equiv \epsilon/2\sigma_0 = v_{\text{th}}/c \ll 1$$

$$\frac{\partial \overline{\rho_w}}{\partial \overline{z}} + p\frac{\partial \overline{\rho_w}}{\partial \overline{x}} - \frac{\overline{U}(\overline{x}+\eta/2) - \overline{U}(\overline{x}-\eta/2)}{\eta}\frac{\partial \overline{\rho_w}}{\partial p} = 0$$

The thermal spreading among the electron rays causes a loss of infomation in the phase space cells of the order of emittance

$$\Rightarrow \quad \frac{\overline{U}(\overline{x}+\eta/2) - \overline{U}(\overline{x}-\eta/2)}{i\eta} i \frac{\partial \overline{\rho_w}}{\partial p} \approx \frac{\overline{U}(\overline{x}+(i\eta/2)\partial/\partial p) - \overline{U}(\overline{x}-(i\eta/2)\partial/\partial p)}{i\eta}\overline{\rho_w}$$

Since we have:

$$\overline{U}(\overline{x}+(i\eta/2)\partial/\partial p) - \overline{U}(\overline{x}-(i\eta/2)\partial/\partial p) = (\partial \overline{U}/\partial \overline{x}) i\eta(\partial/\partial p) + O(\eta^3\partial^3/\partial p^3)$$

the above approximation is equivalent to assume that terms $O(\eta^3\partial^3/\partial p^3)$ are small corrections (semiclassical approximation).

# Landau-type damping and role of the dispersion

R. Fedele, P.K. Shukla, M. Onorato, D. Anderson, M. Lisak, Physics Letters A 303 (2002) 61–66;
R. Fedele, Phil. Trans. R. Soc. A (2008) 366, 1717–1733

Linearization:

$$\rho_w(x,p,s) = \rho_0(p) + \rho_1(x,p,s)$$

$$U(x,s) = U_0 + U_1(x,s) = U_1(x,s)$$

$$\frac{\partial \rho_1}{\partial s} + p\frac{\partial \rho_1}{\partial x} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\left(\frac{\alpha}{2}\right)^{2n} \frac{\partial^{2n+1} U_1}{\partial x^{2n+1}} \rho_0^{(2n+1)}$$

$$\boxed{1 = i\,Z\alpha \int_{-\infty}^{\infty} \frac{\rho_0(p+\alpha k/2) - \rho_0(p-\alpha k/2)}{\alpha k} \frac{dp}{kp-\omega}}$$

$\alpha k << 1$ ➡ $\dfrac{\rho_0(p+\alpha k/2) - \rho_0(p-\alpha k/2)}{\alpha k} \approx d\rho_0/dp \equiv \rho_0^{'}$

Week Landau damping $\boxed{1 = i\alpha Z(k,\omega) \int_{-\infty}^{\infty} \frac{\rho_0^{'}}{kp-\omega}\, dp}$

# *Thanks for your attention!*