Anisotropy in fluid models with full pressure tensor dynamics

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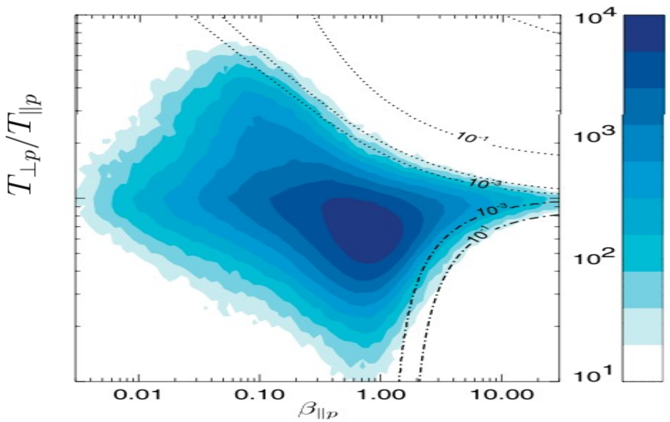
Pressure/temperature anisotropy in collisionless plasmas

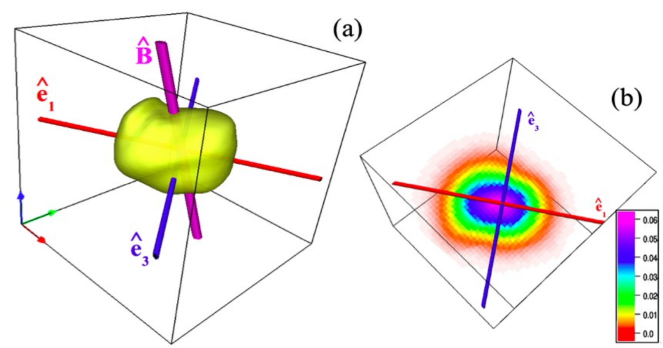
- *Second-order moment anisotropies*, play a prominent role in many low-collision plasma processes (pressure-driven instabilities, magnetic reconnection, turbulence).



[P. Hellinger et al., GRL **33**, L09101 (2006)]

$$\boldsymbol{E} + \boldsymbol{U}\times\boldsymbol{B} = d_i\frac{\boldsymbol{J}\times\boldsymbol{B}}{n} + S^{-1}\boldsymbol{J} - \frac{\rho_s^2}{d_i}\frac{\nabla\cdot\boldsymbol{\Pi}_e}{n} + \frac{d_e^2}{n}\left\{\frac{\partial\boldsymbol{J}}{\partial t} + \nabla\cdot\left(\boldsymbol{UJ} + \boldsymbol{JU} - d_i\frac{\boldsymbol{JJ}}{n}\right)\right\}$$

See, e.g., [H.-J. Cai et al., PoP **4**, 509 (1997)]



[S.Servidio et al., PRL, **108**, 045001 (2012)]

- In most cases *the origin itself of second order moment anisotropy is still debated.*

- *Sometimes* (e.g. solar wind) pressure anisotropy appear to be *correlated with flows*.

Outline

- **Full pressure tensor evolution :**
  - *i) model equation*
  - *ii) kinematics and role of the fluid strain*

- **Gyrotropic and non-gyrotropic anisotropy**

- **Fluid strain as a source of pressure anisotropization :**
  - *i) equation for the evolution of the agyrotropy*
  - *ii) eigenmode analysis (at fixed fluid and magnetic fields)*

- **Example : shear-driven ion anisotropization :**
  - *i) extended Hall-MHD model with cold mass-less electrons*
  - *ii) numerical results*
  - *iii) normal modes of the system and role in the evolution of the anisotropy*
  - *iv) anisotropic equilibrium solutions*
  - *v) implications for turbulence*

- **Fluid description of Weibel-type instabilities :**
  - *i) model*
  - *ii) a few insights from the fluid description*

- **A few remarks on the rôle of the heat flux**

- **Summary and conclusions**

Mathematical model

- Taking the second anisotropic moment from Vlasov equation :

$$\int [\text{Vlasov eq.}]\,(v_i v_j .. v_n) d^3 v$$

$$\underset{n=2}{\Pi_{ij} \equiv nm\langle (v_i - u_i)(v_j - u_j)\rangle} \qquad \underset{n=3}{Q_{ijk} \equiv nm\langle (v_i - u_i)(v_j - u_j)(v_k - u_k)\rangle}$$

$$\frac{\partial}{\partial t}\Pi_{ij} + \overbrace{u_k \frac{\partial}{\partial x_k}\Pi_{ij}}^{\textit{advection}} + \overbrace{\Pi_{ij}\frac{\partial u_k}{\partial x_k}}^{\textit{compression}} + \overbrace{\frac{\partial u_i}{\partial x_k}\Pi_{kj} + \frac{\partial u_j}{\partial x_k}\Pi_{ik}}^{\textit{strain}}$$

$$+ \underbrace{\frac{\partial}{\partial x_k} Q_{ijk}}_{\substack{\textit{gradient of the triadic} \\ \textit{heat-flux tensor}}} = \frac{q}{mc}\underbrace{\left(\varepsilon_{ilm}\Pi_{lj}B_m + \varepsilon_{jlm}B_m\Pi_{il}\right)}_{\textit{gyration due to the magnetic field}}$$

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n=3

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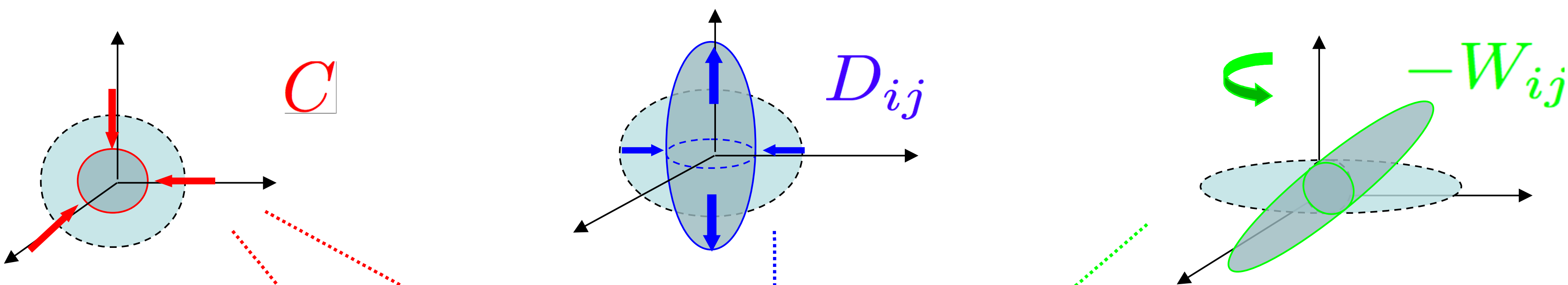
« Kinematic » dynamics of the pressure tensor

- Three kinds of mechanical deformations are possible because of the fluid ***strain*** :

$$\nabla \mathbf{u} = \underbrace{\frac{1}{3}(\nabla\cdot\mathbf{u})\mathbf{I}}_{\substack{\textit{isotropic} \\ \textit{compression}}} + \underbrace{\left[\frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right) - \frac{1}{3}(\nabla\cdot\mathbf{u})\mathbf{I}\right]}_{\substack{\textit{compressionless deformation without} \\ \textit{rotation} \quad (\textbf{\textit{rate of shear}})}} + \underbrace{\frac{1}{2}\left(\nabla\mathbf{u} - (\nabla\mathbf{u})^T\right)}_{\substack{\textit{rotation} \\ (\textbf{\textit{vorticity}})}}$$



$$\partial_t \Pi_{ij} + u_k \partial_k \Pi_{ij} + \overbrace{\Pi_{ij}\partial_k u_k} + \overbrace{\partial_i u_k \Pi_{kj} + \Pi_{ik}\partial_k u_j} = \Omega\left(\varepsilon_{ilm} b_m \Pi_{lj} + \varepsilon_{jlm} b_m \Pi_{il}\right)$$

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$$\frac{d}{dt}\mathbf{\Pi} = -5C\mathbf{\Pi} - \{\mathbf{D}, \mathbf{\Pi}\} + [\mathbf{W} + \mathbf{B}, \mathbf{\Pi}]$$

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(depending on the sign of $\boldsymbol{\omega}\cdot\boldsymbol{B}$)

Anisotropization

$|\omega| \sim 2\Omega$ *resonances*

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Anisotropization

$|\omega|\sim 2\Omega$ *resonances*

- The *traceless strain **D** can modify the internal energy of the plasma* $\mathrm{tr}\{\mathbf{\Pi}\}/2$ *independently from isotropic compressions :*

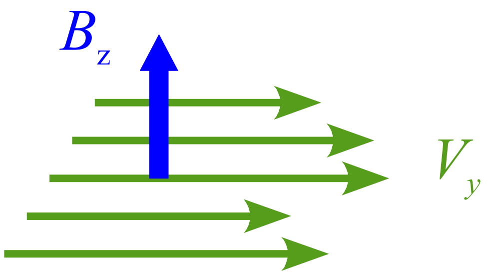
$$\boxed{\frac{d}{dt}\mathrm{tr}\{\mathbf{\Pi}\} = -2\,\mathrm{tr}\{\mathbf{D\Pi}\} + 5C\,\mathrm{tr}\{\mathbf{\Pi}\}}$$

Gyrotropic and non-gyrotropic anisotropy (agyrotropy)

- The commutator term [**W**+**B**, **Π**] mixes the components of the pressure tensor by contributing to gyrotropy in the plane of rotation.

- For simplicity we assume $\boldsymbol{\omega} \parallel \boldsymbol{B} \parallel \boldsymbol{e}_z$.

Ex. :



- We refer the pressure tensor in the perpendicular plane to its principal axes

$$\hat{A}^{gyr} \equiv \frac{\Pi_{||}}{(\Pi_1 + \Pi_2)/2}$$

Gyrotropic anisotropy

$$\hat{A}^{n.g.} \equiv \frac{\Pi_1 - \Pi_2}{\Pi_1 + \Pi_2}$$

Agyrotropy (non-gyrotropic anisotropy)

$$\begin{pmatrix} \Pi_{xx} & \Pi_{xy} & * \\ \Pi_{xy} & \Pi_{yy} & * \\ * & * & \Pi_{zz} \end{pmatrix} \xrightarrow[R_z(\theta)]{} \begin{pmatrix} \Pi_1 & 0 & * \\ 0 & \Pi_2 & * \\ * & * & \Pi_{||} \end{pmatrix} = \begin{pmatrix} \hat{A}^{n.g} + 1 & 0 & * \\ 0 & -\hat{A}^{n.g.} + 1 & * \\ * & * & \hat{A}^{gyr} \end{pmatrix} \frac{\mathrm{tr}\{\boldsymbol{\Pi}_\perp\}}{2}$$

Fluid strain as a source of pressure anisotropization

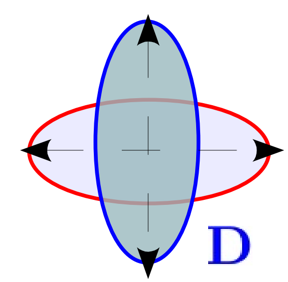
- Assuming a 2D space dependence (∂_z=0) we define : $\hat{A}^{\mathrm{n.g.}} \equiv \frac{2A^{\mathrm{n.g.}}}{\mathrm{tr}\{\mathbf{\Pi}_\perp\}}$

$$\frac{\Pi_{xx} - \Pi_{yy}}{2} = A^{\mathrm{n.g.}} \cos 2\theta \qquad \Pi_{xy} = A^{\mathrm{n.g.}} \sin 2\theta \qquad \frac{D_{xx} - D_{yy}}{2} = D \cos 2\phi \qquad D_{xy} = D \sin 2\phi$$

- An equation for θ and one for $A^{n.g.}$ can be obtained :

$$2\frac{d\theta}{dt} = \underbrace{-(2\Omega_c + \omega_z)}_{(\text{Sign of } \mathbf{B}\cdot\boldsymbol{\omega})} + D\frac{\mathrm{tr}\{\mathbf{\Pi}_\perp\}}{A^{\mathrm{n.g.}}} \sin[2(\theta - \phi)]$$

$$\boxed{\frac{d\hat{A}^{\mathrm{n.g.}}}{dt} = 2D[(\hat{A}^{\mathrm{n.g.}})^2 - 1] \cos[2(\theta - \phi)]}$$



- *$A^{n.g.}$ increases when the principal axes of $\boldsymbol{\Pi}_\perp$ and **D** are dephased by an angle $\pi/4 < \theta < \pi/2$.*

- A slowly varying velocity strain induces a net agyrotropy in the in-plane pressure with an angular shift of $\pi/2$. *The maximum rate of agyrotropy increase is for $\theta - \phi = \pi/2$.*

Fluid strain as a source of pressure anisotropization

- Looking for eigenvalue solutions of the pressure tensor equation at fixed density, velocity and magnetic field with

$$\mathbf{u} = (0, f(x), 0) \qquad \mathbf{B} = (0, 0, B_0)$$

$$\Omega' = f'(x) + \Omega$$

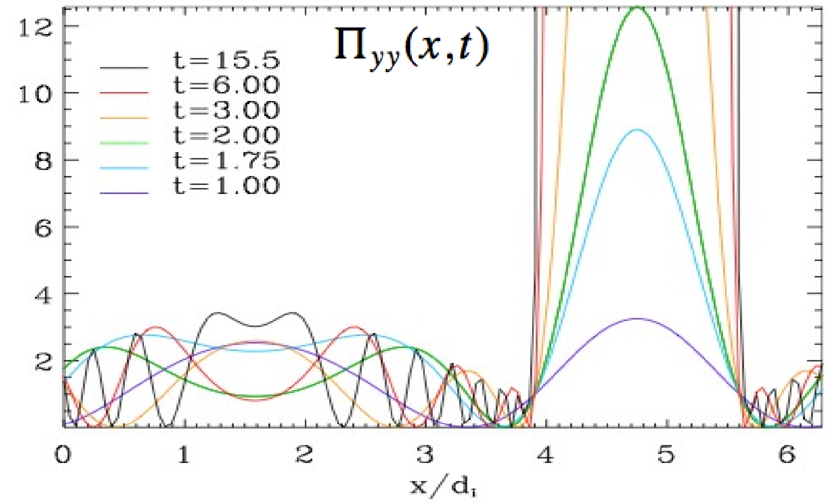
(*The sign of Ω' depends from the sign of* $\boldsymbol{B}\cdot\boldsymbol{\omega}$*!*)

$$\begin{pmatrix} \dot{\Pi}_{xx} \\ \dot{\Pi}_{xy} \\ \dot{\Pi}_{yy} \end{pmatrix} = \begin{pmatrix} 0 & 2\Omega & 0 \\ -\Omega' & 0 & \Omega \\ 0 & -2\Omega' & 0 \end{pmatrix} \begin{pmatrix} \Pi_{xx} \\ \Pi_{xy} \\ \Pi_{yy} \end{pmatrix}$$

three eigenvectors are found :

$$\begin{pmatrix} \Omega \\ 0 \\ \Omega' \end{pmatrix}_{\gamma_0} \quad \begin{pmatrix} 2\Omega \\ -2i\sqrt{\Omega\Omega'} \\ -2\Omega' \end{pmatrix}_{\gamma_-} \quad \begin{pmatrix} 2\Omega \\ 2i\sqrt{\Omega\Omega'} \\ -2\Omega' \end{pmatrix}_{\gamma_+}$$

$$\gamma_0 = 0 \qquad \gamma_- = -2i\sqrt{\Omega\Omega'} \qquad \gamma_+ = 2i\sqrt{\Omega\Omega'}$$



$\boldsymbol{u} = (0, V_0\cos(x/d_i), 0)$ $\quad V_0=-1.5 \quad B_0=1$

- *Exponential growth of the anisotropy* for $\Omega'(x) < 0$ over $t \sim \tau_H = (kV_0)^{-1}$

$$\boxed{\hat{A}^{n.g.} \sim e^{\gamma_+ t} \xrightarrow[t\to\infty]{} 1}$$

Ex. : shear-driven ion-anisotropy generation

- Consider the set of Hall-MHD equations for cold, mass-less electrons

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n\boldsymbol{u}) \qquad \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = \frac{\boldsymbol{J} \times \boldsymbol{B}}{n m_i c} - \frac{\nabla \cdot \boldsymbol{\Pi}}{n m_i}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left(\frac{\boldsymbol{u} - \boldsymbol{J}/(ne)}{c} \times \boldsymbol{B} \right)$$

$$\frac{\partial \boldsymbol{\Pi}}{\partial t} + \nabla \cdot (\boldsymbol{u}\boldsymbol{\Pi}) + (\nabla \boldsymbol{u}) \cdot \boldsymbol{\Pi} + ((\nabla \boldsymbol{u}) \cdot \boldsymbol{\Pi})^T = \Omega_c(\boldsymbol{\Pi} \times \boldsymbol{b} + \boldsymbol{b} \times \boldsymbol{\Pi})$$

- The shear-driven pressure tensor anisotropisation is ``limited" by the self-consistent evolution of *the plasma* that *conserves the total energy*

$$E^{\mathrm{tot}} = \int dx^3 \left(\frac{n m_i u^2}{2} + \frac{B^2}{8\pi} + \frac{\mathrm{tr}\{\boldsymbol{\Pi}\}}{2} \right)$$

and *allows the propagation of linear modes* (*magneto-acoustic modes* and *«magneto-elastic » modes* corresponding to m=2 ion-Bersntein waves).

- These influence the time evolution of the spatial inhomogeneities that anisotropically distribute among the pressure tensor components

Ex. : shear-driven ion-anisotropy generation

i) Energy conservation constraints the ”anisotropization instability”

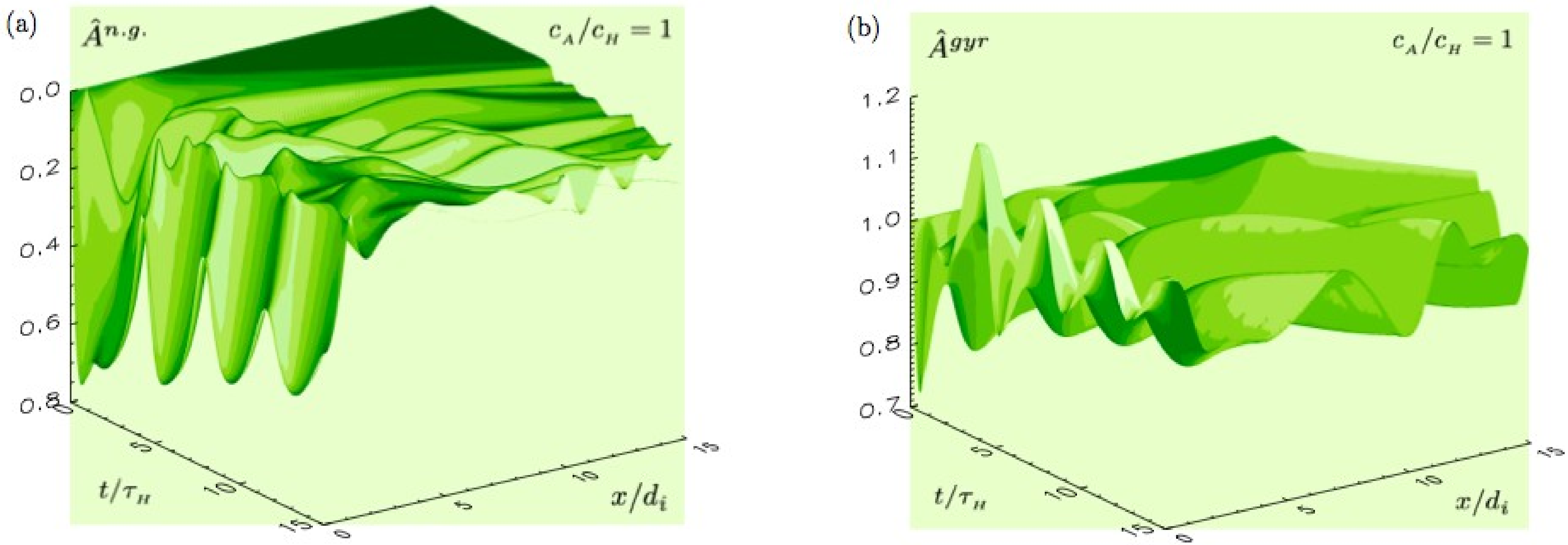
ii) The extent of the shear-induced effects depends from the relative magnitude of the parameter $c_H/(L\Omega)$. *Anisotropization requires $c_H/(L\Omega)$ to be not negligible.*

$$\underbrace{\partial_t \Pi_{ij}}_{\tau^{-1}} + \underbrace{\Pi_{ij}\partial_k u_k + .. + \Pi_{kj}\partial_k u_i}_{c_H/L} = \underbrace{\Omega(\varepsilon_{ilm}\Pi_{lj}b_m + ...)}_{\Omega}$$

iii) The coupling with MHD equations introuces a third parameter.

$$\underbrace{\partial_t \mathbf{u}}_{\tau^{-1}} + \underbrace{\mathbf{u}\cdot\nabla\mathbf{u}}_{\frac{c_H}{L}} = \underbrace{\frac{\mathbf{J}\times\mathbf{B}}{\rho}}_{\frac{c_a}{L}\frac{c_a}{c_H}} - \underbrace{\frac{\nabla\cdot\mathbf{\Pi}}{\rho}}_{\frac{c_s}{L}\frac{c_s}{c_H}} \longrightarrow \left\{ \begin{array}{c} \left(\dfrac{c_a}{c_H}\right)^2 \\ \left(\dfrac{c_s}{c_H}\right)^2 \\ \left(\dfrac{c_a}{c_H}\right)\left(\dfrac{L_H}{d_i}\right) \end{array} \right.$$

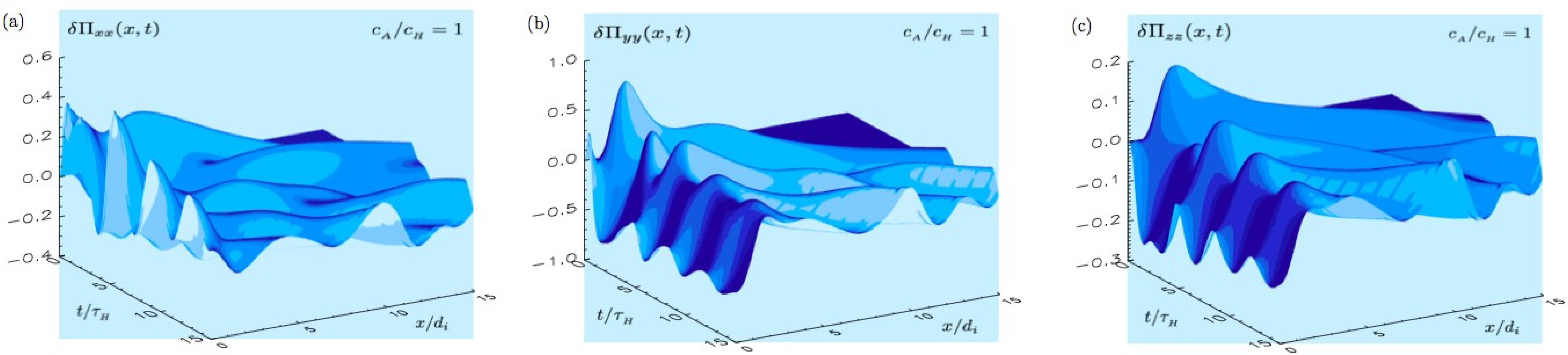
Ex. : shear-driven ion-anisotropy generation



$$\hat{A}^{n.g.} \equiv \frac{\Pi_1 - \Pi_2}{\Pi_1 + \Pi_2}$$

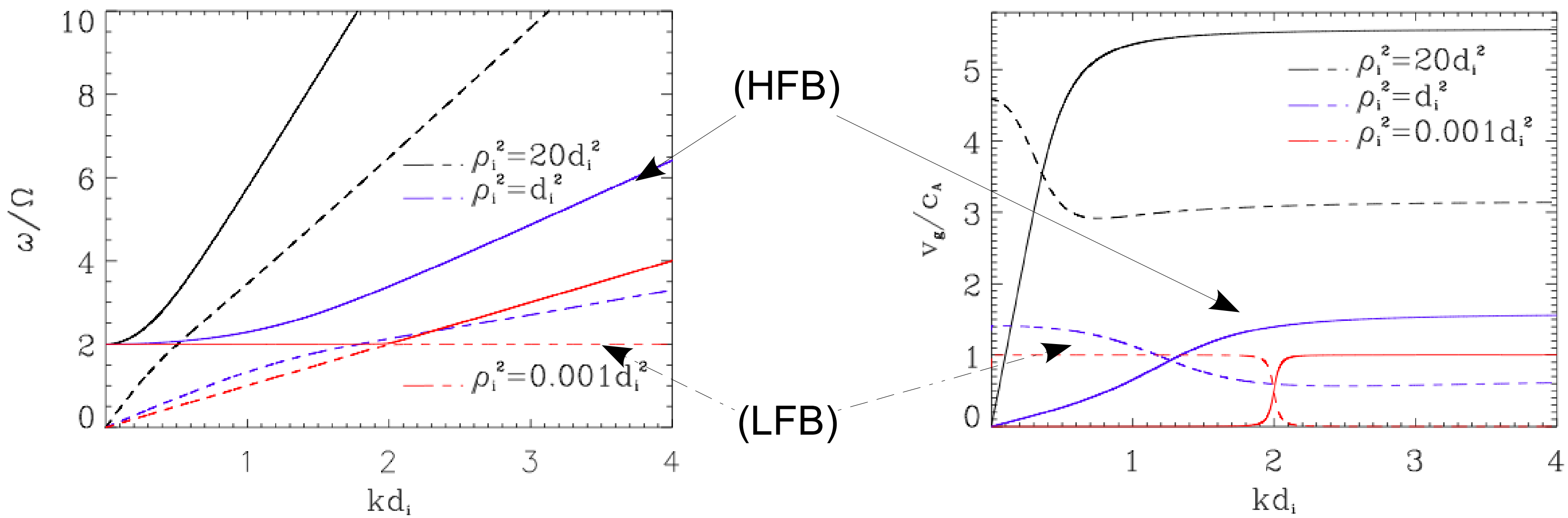
$$u_y(x,0) = \frac{\tanh(x/d_i)}{\cosh^2(x/d_i)}$$

$$\hat{A}^{gyr} \equiv \frac{\Pi_{||}}{(\Pi_1 + \Pi_2)/2}$$



Shear-driven ion-anisotropy generation : normal modes

- Two branches are excited at propagation perpendicular to a uniform magnetic field :



- Lower branch (LFB) → consistent with the full-kinetic magnetosonic wave.

- Higher branch (HFB) → consistent with the ion-Bernstein *m=2* mode, but for the dispersion at small kd_i (the error, numerically negligible by comparison with Vlasov for small $k\rho_i$, does not influence the argument about the anisotropisation).

Fluid + tensor : $\omega_l^2 \sim k^2(c_A^2 + 2c_\perp^2)$ $\omega_h^2 \sim 4\Omega^2 \boxed{+} 2k^2c_\perp^2$

$k \to 0$

Vlasov-Maxwell : $\omega_l^2 \sim k^2(c_A^2 + 2c_\perp^2)$ $\omega_h^2 \sim 4\Omega^2 \boxed{-} 2k^2c_\perp^2$

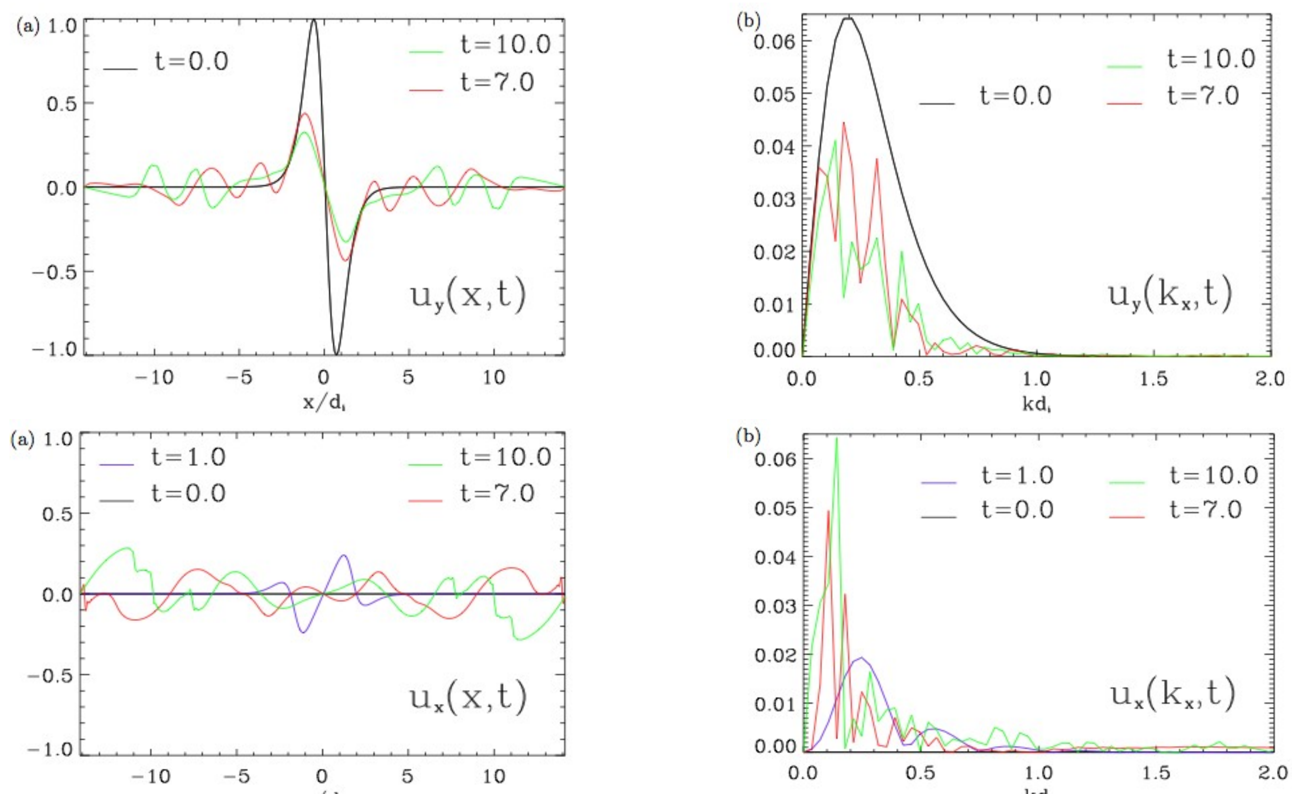
Shear-driven ion-anisotropy generation : normal modes

- Initial perturbation $\boldsymbol{u}$ =(0, $u_y(x)$, 0) <=> superposition of the two branches with equal and opposite u_x amplitudes

$$u_y(x,0) = \frac{\tanh(x/d_i)}{\cosh^2(x/d_i)}$$

$$\frac{\omega_{l,h}}{\Omega} \sim kd_i \ll 1$$

$$\boxed{\begin{pmatrix} u_x \\ u_y \end{pmatrix}_h \sim \begin{pmatrix} 1 \\ -i \end{pmatrix} \qquad \begin{pmatrix} u_x \\ u_y \end{pmatrix}_l \sim \begin{pmatrix} 1 \\ -i\; o(kd_i) \end{pmatrix}}$$



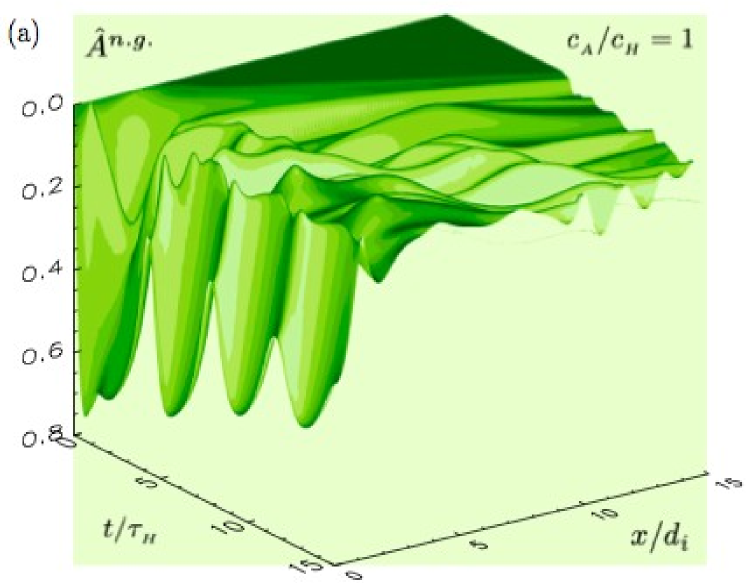
- The time evolution of $u^0_y(x)$ → mainly determined by the HFB : $v_{g,h} \sim (kd_i)c_\perp^{\,2}/c_A$

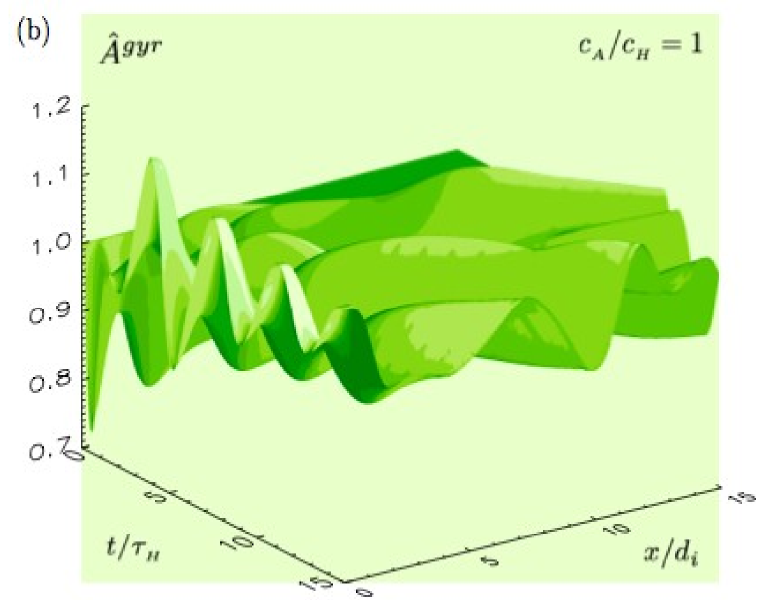
- Both the LFB and HFB contribute to the evolution of $u_x(x)$, where the initial cancellation is removed as time evolves with the LFB component propagating outwards.

Shear-driven ion-anisotropy generation

- *The anisotropization induced by a velocity shear with a spectral distribution at $kd_i << 1$ occurs in a time $\sim \tau_H$ and persists over a time $\sim c_A/(kc_\perp^2)$.*

- For $\tau_H/\tau_B = 1$, the initial agyrotropy is generated over a time scale $\sim L_H/c_H$. Only a fraction ($< kd_i$) of the initial perturbation $u^0_y(x)$ is redistributed by the LFB on the characteristic Alfvén time of the configuration, while the HFB takes a time $d_i/v_{g,h} \sim c_A/(kc_\perp^2) \gg d_i/c_A = \tau_B$ to displace the initial velocity profile by a distance equal to its characteristic size, d_i.





Ex. : shear-driven ion-anisotropy generation at L~ d_i

- Steady solutions of the full pressure tensor equation,

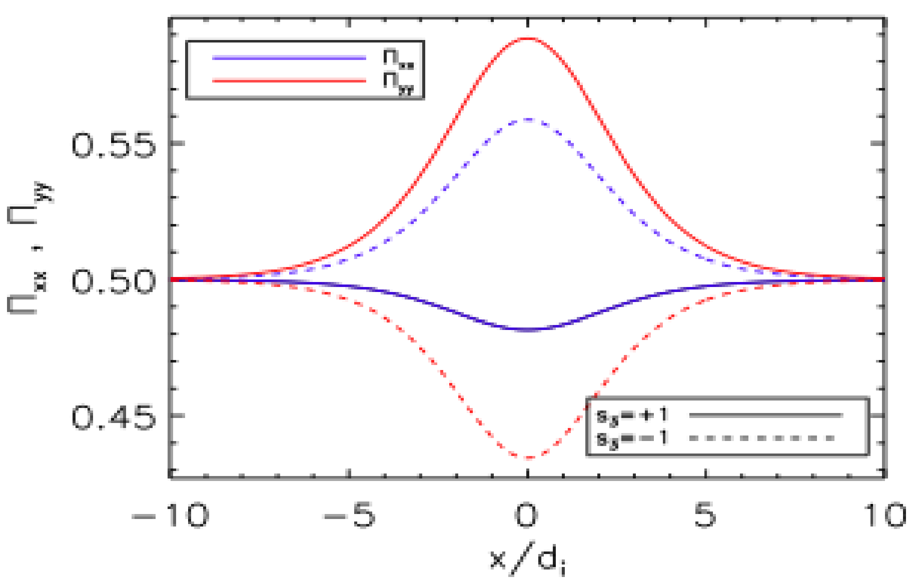
$$
\begin{cases}
\Pi_{\alpha,zz} = p_{\alpha,||} \\
\Pi_{\alpha,xx} = \left(1 - \dfrac{a_\alpha(x)}{1+a_\alpha(x)}\right) p_{\alpha,\perp} \\
\Pi_{\alpha,yy} = \left(1 + \dfrac{a_\alpha(x)}{1+a_\alpha(x)}\right) p_{\alpha,\perp}
\end{cases}
$$

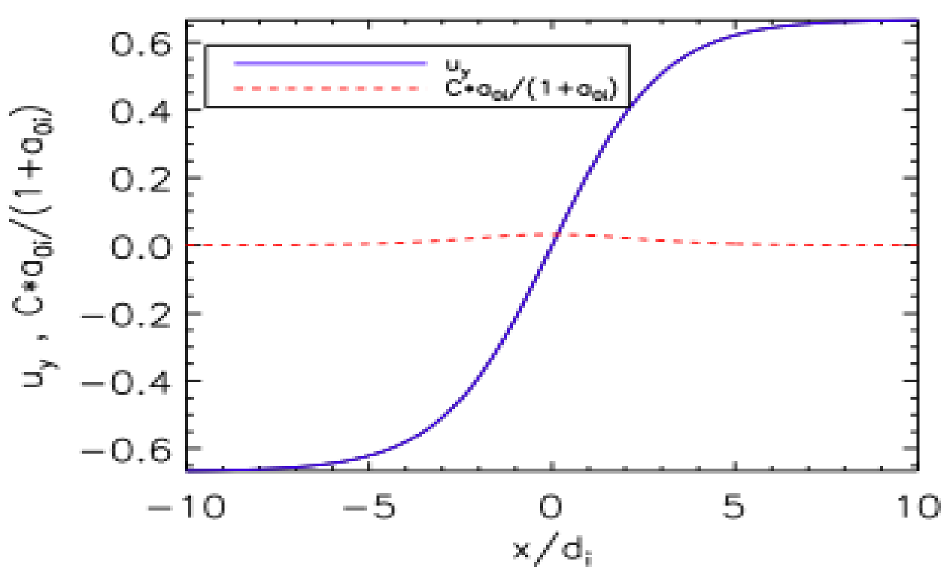
$$
a_\alpha(x) \equiv \mathrm{sign}(\omega \cdot \mathbf{B}) \frac{1}{2} \frac{q_\alpha}{|q_\alpha|} \left(\frac{\partial u_y^0}{\partial x} \right)
$$

valid for

$$
a_\alpha(x) \geq -\frac{1}{2} \quad <=> \quad \Omega' = \partial_x u_y^0 + \Omega \geq 0
$$

can be used to obtain equilibrium configurations for the other fluid moments numerically or perturbatively for small anisotropies [*S.S. Cerri et al., PoP 2014*].





Shear-driven anisotropy generation in turbulence

- Discrepancies with respect to the CGL closure become important when $\tau_H \Omega_c \sim 1$.
- For $c_H \sim c_A$ (Alfvenic turbulence) *pressure anisotropies can be expected when velocity inhomogeneities are generated in the plane perpendicular to* ***B*** *at a scale* $L_H \sim d_i$.

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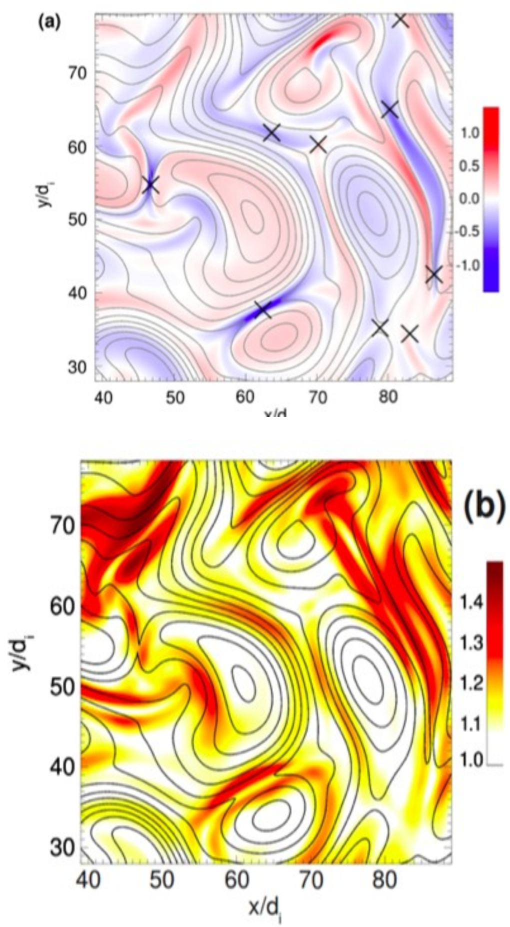
From the parameters of [Servidio, 2012]:

$$c_H \sim c_{A,\perp} \sim \alpha c_A$$

$$\alpha \simeq 0.38,\ 0.58$$

then

$$\boxed{\frac{c_H}{L\Omega} \simeq \alpha \frac{d_i}{L}}$$



From [*S.Servidio et al., PRL,* ***108****, 045001 (2012)*]

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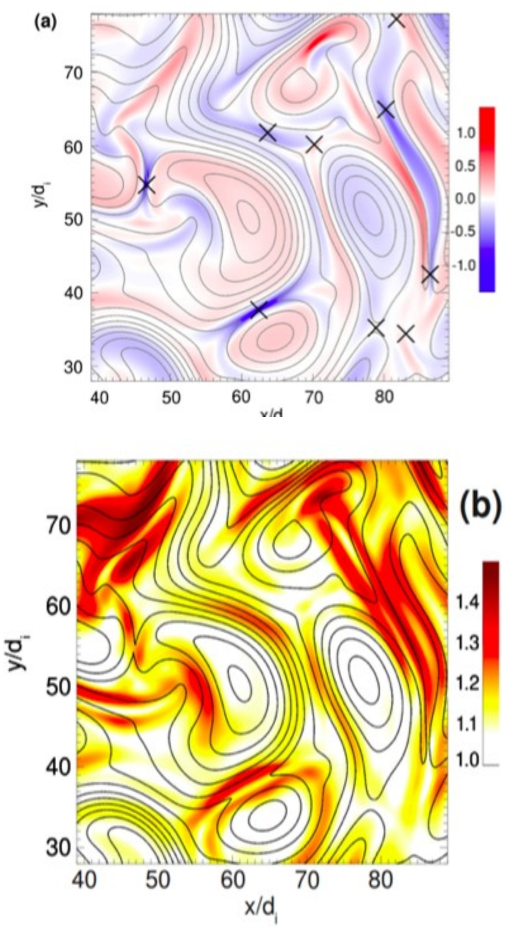

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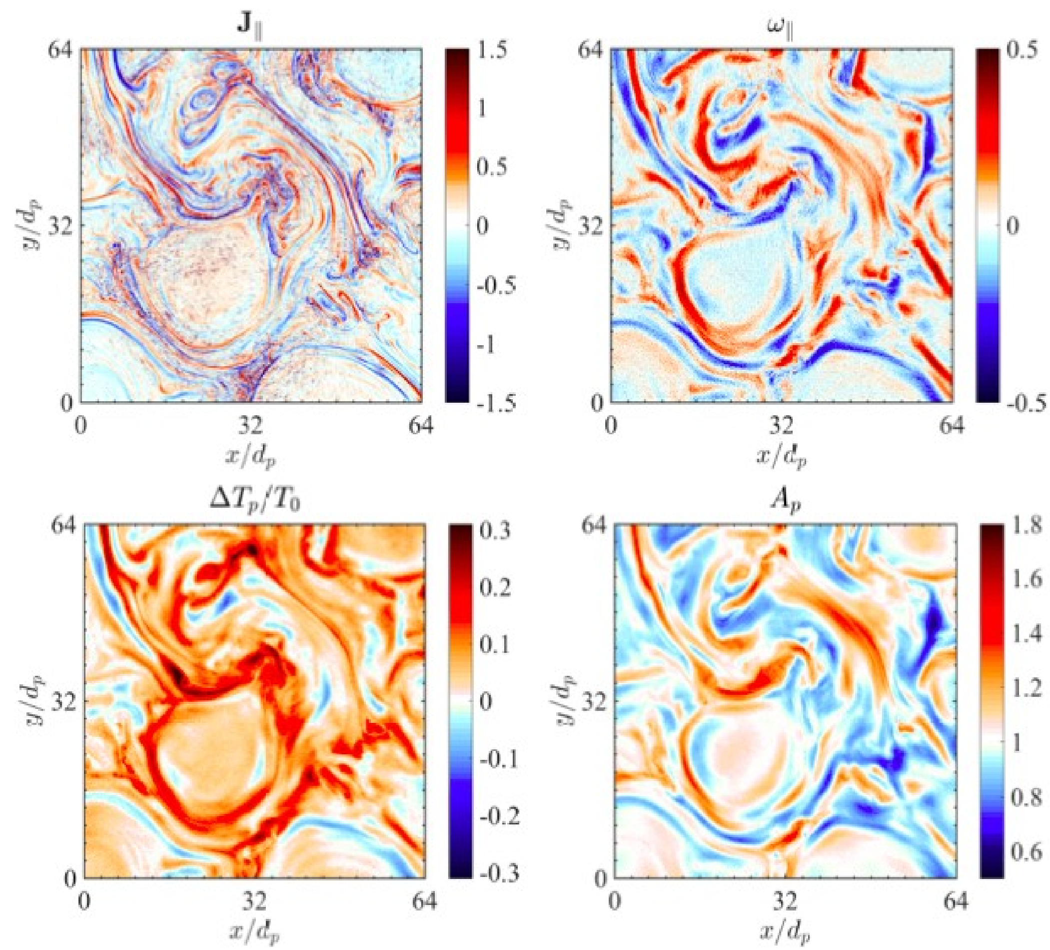


From [*L. Franci et al.,APS Proc.*, ***1720***, *040003 (2016)*]

Fluid description of Weibel-type instabilities

- Consider a configuration of two counter-propagating, warm electron beams (label α), possibly initially non-isotropic (bi-Maxwellian), in an neutralising ion background.

$$\partial_t n_\alpha + \boldsymbol{\nabla} \cdot (\boldsymbol{u}_\alpha n_\alpha) = 0$$

$$\partial_t \boldsymbol{u}_\alpha + \boldsymbol{u}_\alpha \cdot \boldsymbol{\nabla} \boldsymbol{u}_\alpha = -\frac{e}{mc}(c\boldsymbol{E} + \boldsymbol{u}_\alpha \times \boldsymbol{B}) - \frac{\boldsymbol{\nabla} \cdot \boldsymbol{\Pi}_\alpha}{n_\alpha m}$$

$$\boldsymbol{\Pi}_\alpha^{(0)} = \begin{pmatrix} \Pi^{(0)}_{\alpha,xx} & 0 & 0 \\ 0 & \Pi^{(0)}_{\alpha,yy} & 0 \\ 0 & 0 & \Pi^{(0)}_{\alpha,zz} \end{pmatrix}$$

$$\partial_t \boldsymbol{\Pi}_\alpha + \boldsymbol{\nabla} \cdot (\boldsymbol{u}_\alpha \boldsymbol{\Pi}_\alpha) + \boldsymbol{\nabla} \boldsymbol{u}_\alpha \cdot \boldsymbol{\Pi}_\alpha + (\boldsymbol{\nabla} \boldsymbol{u}_\alpha \cdot \boldsymbol{\Pi}_\alpha)^T =$$
$$-\frac{e}{mc}(\boldsymbol{\Pi}_\alpha \times \boldsymbol{B} + (\boldsymbol{\Pi}_\alpha \times \boldsymbol{B})^T) - \cancel{\boldsymbol{\nabla} \cdot \boldsymbol{Q}_\alpha}$$

$$\partial_t \boldsymbol{E} = c\boldsymbol{\nabla} \times \boldsymbol{B} - 4\pi \boldsymbol{J}$$

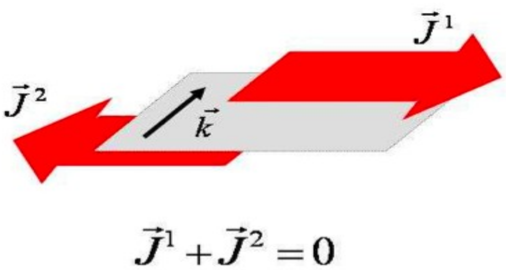
$$\partial_t \boldsymbol{B} = -c\boldsymbol{\nabla} \times \boldsymbol{E}$$

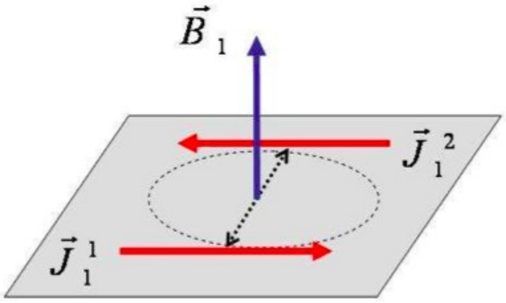
$$\boldsymbol{J} \equiv -e \sum_\alpha n_\alpha \boldsymbol{u}_\alpha$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 4\pi e \left(n_i^0 - \sum_\alpha n_\alpha \right)$$

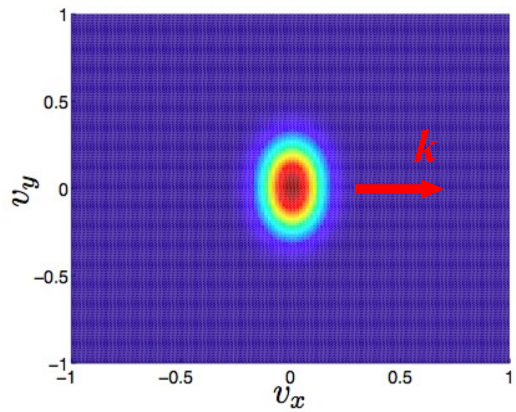
Fluid description of Weibel-type instabilities

- For a null initial, total current density and perpendicular perturbations, the configuration is unstable to the Current Filamentation Instability (CFI) [*Fried, PoF, 1959*].





- On the other hand, a bi-maxwellian electron distribution is unstable to the Weibel Instability (WI) when $T_{\perp_k} > T_{\|k}$ [*Weibel, PRL, 1959*].



- Considering two warm, bi-maxwellian electron beams allows to consider the coupled WI-CFI mode, by extending the full-pressure tensor analysis performed by [*Basu, PoP, 2002*] for the WI only in the strong anisotropy limit, $T_{\perp_k} >> T_{\|k}$.

Fluid description of Weibel-type instabilities

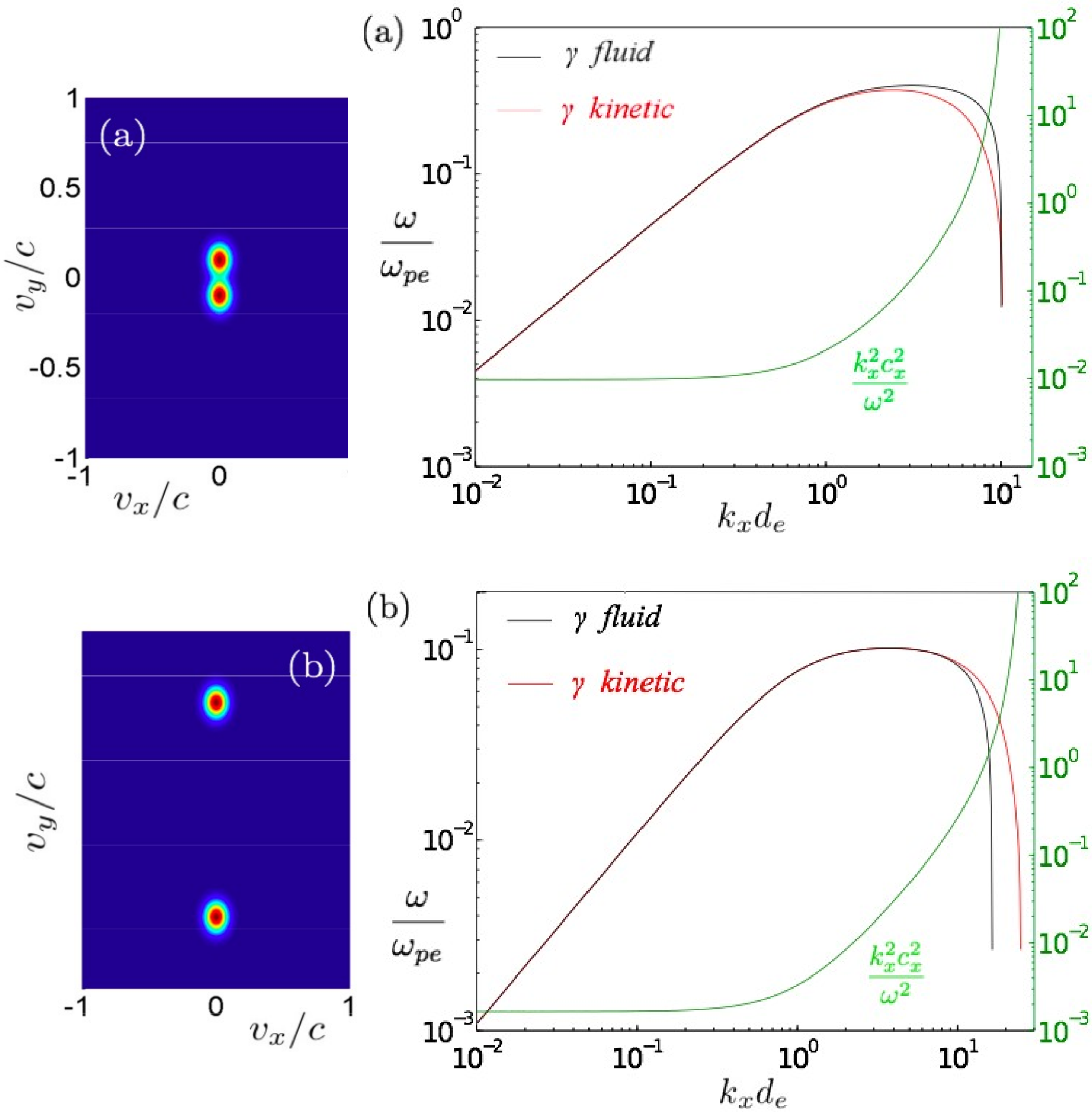
- Linearisation around a homogeneous equilibrium for $\boldsymbol{k}=(k_x, 0, 0)$ gives results in good agreement with the kinetic description in both the WI- and CFI- dominated regimes:

$$c_{x,\alpha}^2 \equiv \frac{\Pi_{xx,\alpha}^{(0)}}{m n_\alpha^{(0)}}$$

$$c_{y,\alpha}^2 \equiv \frac{\Pi_{yy,\alpha}^{(0)}}{m n_\alpha^{(0)}}$$

$$\omega_{pe,\alpha}^2 \equiv \frac{4\pi n_\alpha^{(0)} e^2}{m}$$

$$\omega_{pe}^2 \equiv \omega_{pe,1}^2 + \omega_{pe,2}^2$$



WI-dominated

$$\tilde{\gamma}_{WI} \simeq \omega_{pe} \frac{k_x c_y}{\sqrt{\omega_{pe}^2 + k_x^2 c^2}}$$

$$k_c^{WI} d_e = \sqrt{\frac{c_y^2 - c_x^2}{c_x^2}} = \sqrt{\frac{T_y}{T_x} - 1}$$

CFI-dominated

$$\tilde{\gamma}_{CFI} \simeq \omega_{pe} \frac{k_x |u_0|}{\sqrt{\omega_{pe}^2 + k_x^2 c^2}}$$

$$k_c^{CFI} d_e = \frac{|u_0|}{\sqrt{3} c_x}$$

Fluid description of Weibel-type instabilities

- A few non-trivial results of the fluid approach about interpretation of known kinetic results [*M. Sarrat et al., to be submitted 2016*].

  - We can understand in terms dynamical quantities the *role played by thermal features in coupling electrostatic effects to the normally purely e.m. CFI* [*Tzoufras et al., PRL 2006*], [*Bret et al., PoP 2007*]

$$[\boldsymbol{D}] = \begin{pmatrix} D_{xx} & D_{xy} & 0 \\ D_{xy} & D_{yy} & 0 \\ 0 & 0 & D_{zz} \end{pmatrix}$$

$$D_{xy} = 0 \qquad \textit{for} \qquad \Pi^{(0)}_{xx,1} u^{(0)}_{y,1} + \Pi^{(0)}_{xx,2} u^{(0)}_{y,2} = 0$$

$$[\boldsymbol{D}] \cdot \boldsymbol{E} \equiv 0$$

$$\left(n^{(0)}_1 u^{(0)}_{y,1} + n^{(0)}_2 u^{(0)}_{y,2} = 0 \right)$$

  - In the case of *purely symmetric beams the coupled WI-CFI* essentially *consists of a superposition of the two WI and CFI pure modes*

$$\gamma_{WI/CFI} \simeq \omega_{pe} \sqrt{\frac{k_x^2 (c_y^2 + u_0^2)}{\omega_{pe}^2 + k_x^2 c^2}} = \sqrt{\tilde{\gamma}^2_{WI} + \tilde{\gamma}^2_{CFI}} \qquad k_c = \frac{\omega_{pe}}{c} \sqrt{\frac{c_y^2 - c_x^2}{c_x^2} + \frac{u_0^2}{3c_x^2}}$$

Fluid description of Weibel-type instabilities

- It easily understood how pressure anisotropy make it possible the propagation of low-frequency, transverse e.m. waves, which would otherwise be damped in an istropic plasma

$$\omega_{pe}^2 \gg k_x^2 c_x^2$$

$$N \equiv \frac{kc}{\omega} = \sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}\left(\frac{\omega^2 - k_x^2(c_y^2 - c_x^2)}{\omega^2 - k_x^2 c_x^2}\right)}$$

$N^2 > 0$ also for $\omega^2 < k_x^2(c_y^2 - c_x^2)$

thanks to $\dfrac{\tilde{\Pi}_{xy}}{mn^{(0)}} = \dfrac{k_x c_x^2}{\omega}\tilde{u}_y - i\dfrac{e}{m}\dfrac{c_y^2 - c_x^2}{\omega}\tilde{B}_z$ $\qquad \tilde{u}_y = -i\dfrac{e}{m\omega}\tilde{E}_y + \dfrac{k_x}{\omega}\dfrac{\tilde{\Pi}_{xy}}{mn^{(0)}}$

$$\partial_t \boldsymbol{\Pi}_\alpha + \boldsymbol{\nabla}\cdot(\boldsymbol{u}_\alpha \boldsymbol{\Pi}_\alpha) + \boldsymbol{\nabla}\boldsymbol{u}_\alpha \cdot \boldsymbol{\Pi}_\alpha + (\boldsymbol{\nabla}\boldsymbol{u}_\alpha \cdot \boldsymbol{\Pi}_\alpha)^T =$$
$$-\frac{e}{mc}\left(\boldsymbol{\Pi}_\alpha \times \boldsymbol{B} + (\boldsymbol{\Pi}_\alpha \times \boldsymbol{B})^T\right) - \cancel{\boldsymbol{\nabla}\cdot \boldsymbol{Q}_\alpha}$$

A few comments on the heat flux

- The closure condition on the heat flux (and of higher order moments) seems to strongly depend on the specific problem considered :

Ex. :

- *Shear-induced generation of ion anisotropy* :
  -) The neglect of div($\boldsymbol{Q}$) seems a priori reasonable for $k_\perp >> k_\parallel$
  -) The full-pressure tensor model *without heat-flux* allows a consistent description of
- the dispersion relation of CGL-FLR corrections to the fast magnetoacoustic mode at perpendicular propagation. The inclusion of the first order heat flux corrections ($\sim k^4 \rho_i^4$) just allows to recover the exact coefficient of the corresponding Vlasov-Maxwell dispersion relation.

- *Pressure tensor description of the Weibel Instability in a fluid model* :
  -) Without div($\boldsymbol{Q}$) a good agreement with the kinetic dispersion relation is obtained, as long as we are not close to the threshold condition (anisotropy approaching 1), also as far as the kinetic cut-off wave-length is concerned.
  -) When the heat-flux correction are included, the cut-off at large wave-number is lost. Contributions related to the fourth order moment should be probably kept into account (see [*P.L. Sulem, T. Passot, JPP 2015*]).

Conclusions

- *Pressure anisotropization may be induced by a velocity shear* through the action of the strain tensor. Anisotropization is at maximum when the major principal axes of the fluid strain and of the pressure tensor are orthogonal one to each other, *occurs over a time scale L/c_H , and the sign of the product $\boldsymbol{\omega}\cdot\mathbf{B}$ and the magnitude of ratio $c_H/(L\Omega)$ rule the process.* [D.Del Sarto et al., PRE (2016)]
- The proposed *mechanism, valid for both ions and electrons*, seems a good candidate for the understanding of the correlation between velocity shear and non-gyrotropic anisotropization and *has implications for plasma turbulence (-THOR measurements!)*. [D. Del Sarto et al., PRE (2016)]
- *Stationary non-gyrotropic solutions can be devised, which strongly depend on the sign of $\boldsymbol{\omega}\cdot\mathbf{B}$* [S.S.Cerri et al., PoP 2014].
- In the case of ions, the anisotropization induced by a velocity shear with spectral distribution at $kd_i << 1$ occurs in a time $\sim \tau_H$ and persists over a time $\sim c_A/(kc_\perp^2)$, due to the interplay with the normal modes that propagate in the plasma [D. Del Sarto et al., PRE 2016; arxive:1509.04938]
- A fluid description of Weibel instabilities is made possible by including the full-pressure tensor evolution, which allows a better insight on different features of the instability [M. Sarrat et al., to be submitted].

References

Full pressure tensor evolution :

D. Del Sarto, F. Pegoraro, F. Califano *Pressure anisotropy and small spatial scales induced by a velocity shear*, Phys. Rev. E **93**, 053203 (2016).

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S.S. Cerri, F. Pegoraro, F. Califano, D. Del Sarto, F. Jenko, *Pressure tensor in the presence of velocity shear : stationary solutions and self-consistent equilibria*, Physics of Plasmas **21**, 112109 (2014).

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