Pulsar magnetospheres and winds

A challenge for plasma physicists and astrophysicists

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Summary

What is a pulsar ?

1. neutron star
   compact object ⇒ strong gravity effects

$$\xi \equiv \frac{G\,M}{R\,c^2} \approx 0.35$$

2. strongly magnetized
   ⇒ plasmas, QED effects (pair creation)

$$B_q \equiv \frac{m^2\,c^2}{e\,\hbar} \approx 4.4 \times 10^9 \ \mathrm{T}$$

3. rotating
   ⇒ huge electric fields

$$E_{\mathrm{schw}} \equiv c\,B_q \approx 1.3 \times 10^{18} \mathrm{V/m}$$

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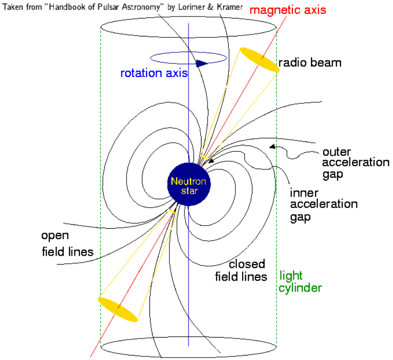

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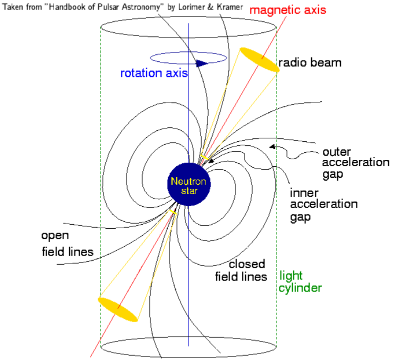


Some useful definitions

- obliquity χ : angle between magnetic moment $\vec{\mu}$ and rotation axis $\vec{\Omega}$
- aligned/perpendicular/oblique rotator : $\chi = 0/90^o/$any value
- light cylinder radius : surface on which a particle corotating with the neutron star reaches the speed of light c : $r_{\rm L} = c/\Omega$
  $\Rightarrow$ transition from quasi-static to wave zone ($\Rightarrow$ very different plasma regimes)

Neutron star magnetospheres : orders of magnitude

- period $P \in [1\ ms, 1\ s]$
- period derivative $\dot{P} \in [10^{-18}, 10^{-15}]$

$\Rightarrow$ spin-down losses well constrained

$$L_{\rm sp} = 4\,\pi^2\,I\,\dot{P}\,P^{-3} \approx 10^{24-31}\ W$$

very different from black holes or accreting neutron stars

- inferred magnetic field estimate by dipole radiation

$$B = 3.2 \times 10^{15}\,\sqrt{P\,\dot{P}} = 10^{5-8}\ T$$

- $\Rightarrow$ consistent with magnetic flux conservation during gravitational collapse
- but no constrain on the geometry (obliquity χ)
- probably not a good guess if multipoles present.

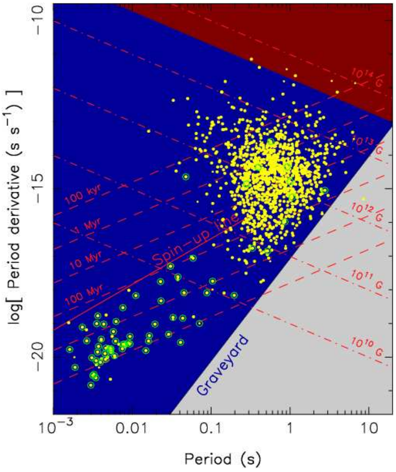


FIGURE : $P - \dot{P}$ diagramm.

Pulsar magnetosphere : orders of magnitude

Electromagnetic and gravitational field characteristics

- electric field induced at the stellar crust

$$E = \Omega\, B\, R = 10^{13}\ \mathrm{V/m}$$

  $\Rightarrow$ instantaneous acceleration at ultra-relativistic speeds, Lorentz factor $\gamma \gg 1$ ($\tau_{\mathrm{acc}} < 10^{-20}$ s)
- negligible gravitational force for protons ! ! !

$$\frac{F_{\mathrm{grav}}}{F_{\mathrm{em}}} \approx \frac{G\, M\, m_p / R^2}{e\, \Omega\, B\, R} \approx 10^{-12} \ll 1$$

even smaller for electrons/positrons (m_e/m_p).

$\Rightarrow$ dynamic of the magnetosphere dominated by the electromagnetic field

Neutron star characteristics

- masse $M \approx 1.4\, M_{\odot}$.
- radius $R \approx 10$ km.
- centrale density $\rho_c \approx 10^{17}\ \mathrm{kg/m^3}$.

Pulsar magnetosphere : the challenges

Quantity	Estimation	Second	Millisecond
Rotation frequency (Hz)	$\nu_* = \frac{1}{P}$	1	1.000
Luminosity (W)	$L = 4\,\pi^2\,I\,\dot{P}\,P^{-3}$	6.3×10^{24}	6.3×10^{30}
Magnetic field (T)	$B = \sqrt{\frac{3\,\mu_0\,c^3}{32\,\pi^3}}\,\frac{\sqrt{I\,P\,\dot{P}}}{R^3}$	7.4×10^{7}	7.4×10^{4}
Electric field (V/m)	$E = \Omega\,B\,R$	7.5×10^{12}	7.5×10^{12}
Gavitational/electric force	$\frac{G\,M\,m_e}{R^2\,e\,E}$	9.7×10^{-12}	9.7×10^{-12}
Light cylinder radius (km)	$r_L = \frac{c}{\Omega}$	47 700	47.7
Particle number density at R (m^{-3})	$n = 2\,\varepsilon_0\,\frac{\Omega\,B}{e}$	6.9×10^{16}	6.9×10^{16}
Particle number density at r_L (m^{-3})		1.1×10^{6}	1.1×10^{15}
Particle flux (s^{-1})	$\mathcal{F} = \frac{4\,\pi\,\varepsilon_0}{e}\,\Omega^2\,B\,R^3$	7.5×10^{29}	7.5×10^{32}
Plasma frequency at R (Hz)	$\nu_p = \frac{1}{2\,\pi}\sqrt{\frac{n\,e^2}{\varepsilon_0\,m_e}}$	2.3×10^{9}	2.3×10^{9}
Plasma frequency at r_L (Hz)		9.4×10^{3}	2.9×10^{8}
Cyclotron frequency at R (Hz)	$\nu_B = \frac{e\,B}{2\,\pi\,m_e}$	2.8×10^{18}	2.8×10^{15}
Cyclotron frequency at r_L (Hz)		4.5×10^{7}	4.5×10^{13}
Characteristic age (years)	$\tau = \frac{P}{2\,\dot{P}}$	1.6×10^{7}	1.6×10^{7}
Gravitational potential energy (J)	$E_g = \frac{3}{5}\,\frac{G\,M^2}{R}$	2.6×10^{46}	2.6×10^{46}
Rotational kinetic energy (J)	$E_k = \frac{1}{2}\,I\,\Omega^2$	3.2×10^{39}	3.2×10^{45}
Magnetic energy (J)	$E_B = \frac{4\,\pi}{3}\,\frac{B^2\,R^3}{2\,\mu_0}$	1.62×10^{34}	1.62×10^{28}
Thermal energy (J)	$E_{th} = \frac{3}{2}\,N\,k\,T$	3.4×10^{40}	3.4×10^{40}

TABLE : The fundamental parameters of a normal and a millisecond pulsar.

Exact solutions

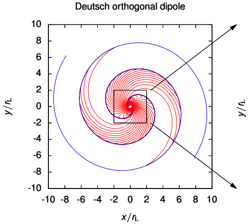
- exact analytical solution for a rotating dipole in vacuum (Deutsch, 1955)
- spindown power due to magnetodipole losses. For a oblique rotator

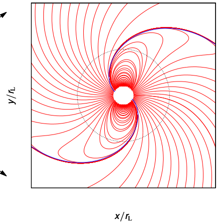
$$L_\perp^{\rm vac} = \frac{8\,\pi\,B^2\,\Omega^4\,R^6}{3\,\mu_0\,c^3}\,\sin^2\chi$$

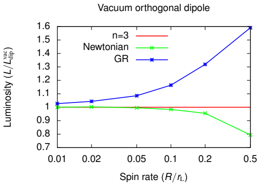
- torque exerted on the surface by charges and currents (Michel & Goldwire, 1970; Davis & Goldstein, 1970)

$\Rightarrow$ secular evolution of the inclination angle

- two singular open field lines leading to a two armed archimedean spiral







- exact analytical solutions for multipoles also exist (Bonazzola et al., 2015; Pétri, 2015)

$\Rightarrow$ useful to enhance the pair production rate at the polar caps (Harding & Muslimov, 2011)

The role of the plasma

- plasma required observationally $\Rightarrow$ broadband radiation detected on Earth.
- particles needed to furnish charges and currents in the magnetosphere.

Analytical study intractable, recent progress via numerical simulations of which most extensively studied

- force-free electrodynamics (FFE or magnetodynamics) : zero mass limit. No energy dissipation.
- resistive magnetodynamics : transfer of energy from field to particles. Prescription not unique. Plasma motion not solved.
- magnetohydrodynamics (MHD) : particle inertia taken into account and the full stress-energy tensor, matter and field, is solved. Ideal and resistive MHD regimes.
- multi-fluids : evolve each species independently, coupling through electromagnetic interactions.
- fully kinetic treatment : individual particle acceleration that are out of thermal equilibrium. Needs to solve the full Vlasov-Maxwell equations.
- radiation reaction limit : acceleration compensated by radiation reaction. Particle motion solved analytically in terms of the external electromagnetic field.

The “standard model” of an ideal pulsar

The full system to solve :

$$\begin{aligned} \nabla_\mu(T^{\mu\nu}_{\rm em} + T^{\mu\nu}_{\rm mat}) &= 0 \\ \nabla_\mu {}^*F^{\mu\nu} &= 0 \\ \nabla_\mu(\rho_{\rm m}\, u^\mu) &= 0 \\ F^{\mu\nu}\, u_\nu &= 0 \end{aligned}$$

Some simplification : force-free magnetosphere ($F^{\mu\nu}\, J_\nu = 0$)

$$\rho_{\rm e}\,\vec{E} + \vec{j} \wedge \vec{B} = \vec{0}$$

- magnetic energy density $\frac{B^2}{2\,\mu_0} \gg$ any other energy densities.
- particle inertia neglected : zero mass limit.
- no dissipation : ideal MHD

$$\vec{E} + \vec{v} \wedge \vec{B} = \vec{0}$$

- no pressure : cold plasma.

Force-free magnetospheres

Simplest approach to pulsar electrodynamics
ideal MHD without particle inertia and without radiation

- Maxwell equations

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{\sqrt{\gamma}} \partial_t(\sqrt{\gamma}\,\mathbf{B})$$

$$\nabla \cdot \mathbf{D} = \rho_{\rm e}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{1}{\sqrt{\gamma}} \partial_t(\sqrt{\gamma}\,\mathbf{D})$$

- FFE current prescription (constraints $\mathbf{E} \cdot \mathbf{B} = 0$ and $E < c\,B$)

$$\mathbf{J} = \rho_{\rm e} \frac{\mathbf{E} \wedge \mathbf{B}}{B^2} + \frac{\mathbf{B} \cdot \nabla \times \mathbf{B}/\mu_0 - \varepsilon_0\,\mathbf{E} \cdot \nabla \times \mathbf{E}}{B^2}\,\mathbf{B}$$

$$\rho_{\rm e} = \varepsilon_0\,\nabla \cdot \mathbf{E}$$

No fluid quantity enters into the system to be solved. (Spitkovsky, 2006; Komissarov, 2006; McKinney, 2006; Pétri, 2012; Paschalidis & Shapiro, 2013; Cao et al., 2016)

Force-free magnetospheres

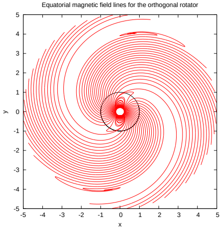


FIGURE : *Magnetic field of the perpendicular rotator* $\chi = 90^o$.

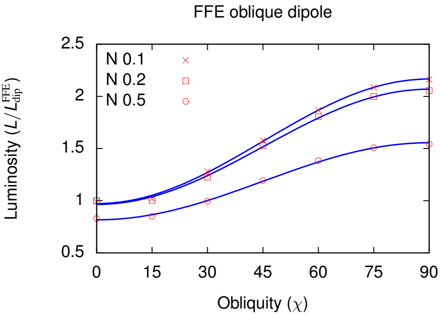


FIGURE : *Spin-down luminosity vs* χ *from simulations in red and fit in blue.*

Plasma filled magnetosphere spindown

$$L_{\rm sp}^{FFE} \approx \frac{3}{2} L_{\perp}^{\rm vac} (1 + \sin^2 \chi)$$

to be compared with vacuum

$$L_{\rm sp}^{vac} \approx L_{\perp}^{\rm vac} \sin^2 \chi$$

(Spitkovsky, 2006; Pétri, 2012)

Includes particle inertia but not particle acceleration

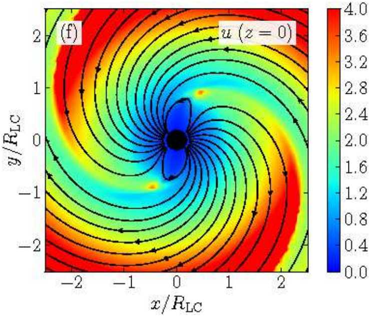


FIGURE : *Perpendicular rotator* $\chi = 90^o$

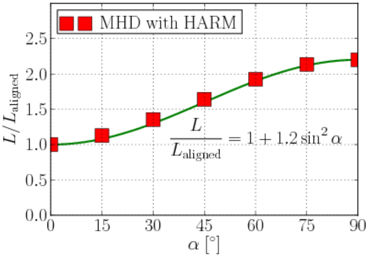


FIGURE : *Spin-down luminosity vs obliquity* χ.

(Tchekhovskoy et al., 2013)

Includes particle inertia AND particle acceleration self-consistently

Equation of motion for a particle (Lorentz force)

$$m \frac{du^\alpha}{d\tau} = q \, F^{\alpha \, \mu} \, u_\mu$$

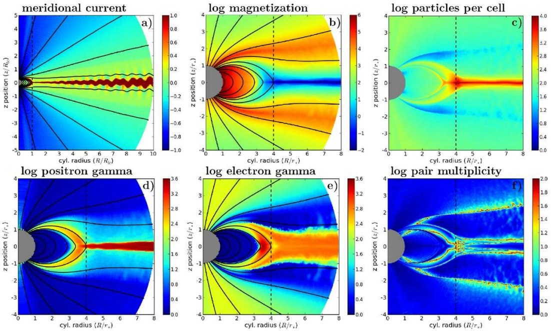


FIGURE : *Plasma properties*

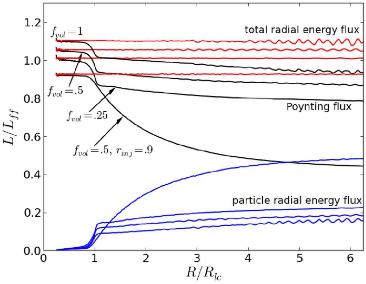


FIGURE : *Particle and Poynting energy flux*

(Belyaev, 2015)

Equation of motion for a particle (Lorentz force + radiation reaction)

$$m \frac{du^\alpha}{d\tau} = q F^{\alpha\,\mu} u_\mu + g^\alpha$$

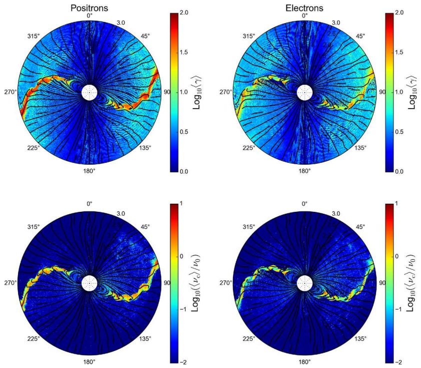


FIGURE : *Lorentz factor and characteristics synchrophoton frequency.*

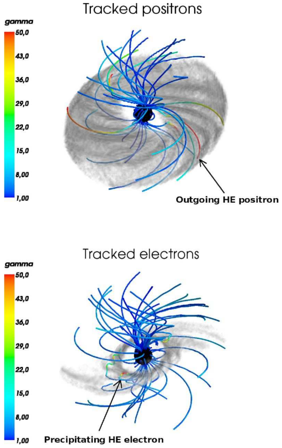


FIGURE : *Electron and positron trajectories.*

Radiation reaction limit

Radiation back reacts on particle motion, a friction appears and at equilibrium

$$q\,(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}) = K\,\mathbf{v}$$

where K represents the intensity of emission.
For ultra relativistic motion, analytical solutions exist

$$K^2 \approx \frac{q^2}{2\,c^2}\left[E^2 - c^2\,B^2 \pm \sqrt{(E^2 - c^2\,B^2)^2 + 4\,c^2\,(\mathbf{E}\cdot\mathbf{B})^2}\right]$$

The equation of motion becomes an algebraic equation for $\mathbf{v}$ depending solely on the electromagnetic field $\mathbf{E}, \mathbf{B}$

$$(K^2 + q^2\,B^2)\,\mathbf{v} = q^2\,\mathbf{E}\wedge\mathbf{B} + q\,K\,\mathbf{E} + q^3\,\frac{\mathbf{E}\cdot\mathbf{B}}{K}\,\mathbf{B}$$

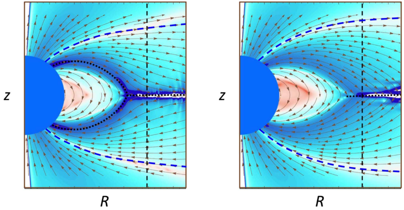


FIGURE : *With (right) and without (left) radiation reaction* (Contopoulos, 2016)

Do we need general relativity ?

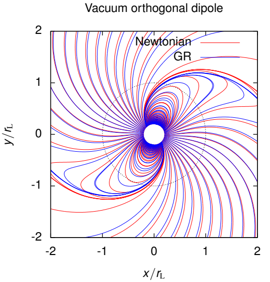


FIGURE : *Magnetic field lines in the equatorial plane for $R/r_L/R = 0.2$.*

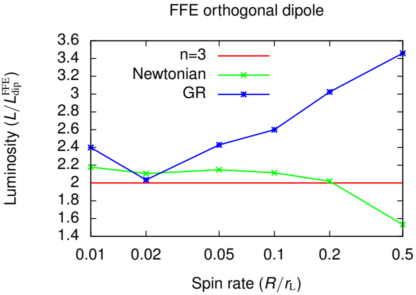


FIGURE : *Spindown luminosity.*

(Pétri, 2016)

YES we need GR for a good quantitative analysis of energy budget an electromagnetic field topology.

Do we need QED on a global scale ?

On small scales obviously yes for pair creation.
The critical magnetic field is

$$B_{\rm qed} = \frac{m_{\rm e}^2\, c^2}{e\, \hbar} \approx 4.4 \times 10^9 \ {\rm T}$$

Maxwell equations become non-linear for $B \gtrsim B_{\rm qed}$.
$\Rightarrow$ Corrections to lowest order by expansion of Euler-Heisenberg Lagrangian post-Mawxellian parameters like post-Newtonian gravity
$\Rightarrow$ Quantum vacuum equivalent to a medium : $\mathbf{D}(\mathbf{E},\mathbf{B}), \mathbf{H}(\mathbf{E},\mathbf{B})$

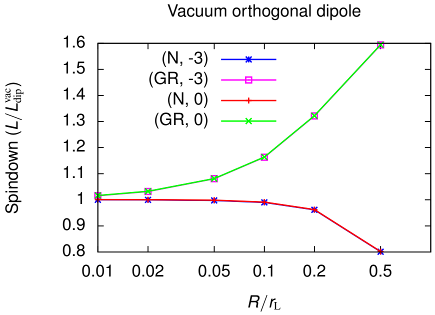


FIGURE : *Spindown luminosity for different rotation rates, magnetic field strengths given by* $\log(B/B_{\rm qed})$ *and gravitational field (Newtonian or GR).*

(Pétri, submitted)

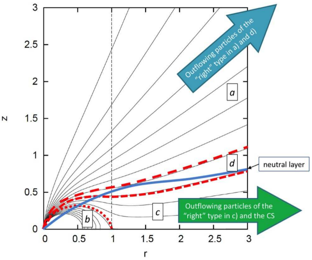
QED does not influence neutron star global vacuum electrodynamics.
Same conclusions for FFE magnetospheres.

Towards general agreements about dense pulsar magnetospheres

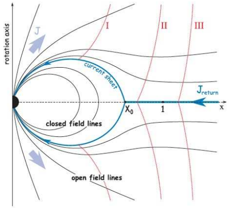
- filled with electron/positron pairs almost everywhere
- FFE approximation satisfactory on a global scale
- formation of an equatorial current sheet
- efficient particle acceleration and emission in this sheet
- Y-point of great importance for the dynamics/spindown losses

$$\dot{E}_Y \approx \left(\frac{r_{\rm L}}{R_Y}\right)^2 \dot{E}_L \geqslant \dot{E}_L$$

  maybe solution for braking index ?
- breakdown of ideal MHD/FFE in some small regions
- magnetic reconnection invoked in these regions and in the sheet



(Contopoulos, 2016)



(Timokhin, 2006)

Conclusions

O Pulsar magnetosphere and wind

- global structure well constrained
- global FFE picture satisfactory
- good agreement between FFE/MHD and PIC simulations
- magnetosphere naturally linked to its striped wind

⚠ Caveats

- some dissipation regions required for emission
- self-consistent acceleration of particles only through PIC/Vlasov simulations
- particle injection rate unknown
- a lot of microphysics still missing
- realistic fully kinetic simulations impossible because

$$\frac{\omega_B}{\Omega} \gtrsim 10^{12} - 10^{18}$$

If you have good ideas to deal numerically with such strong electromagnetic fields (and may be also with radiation) you are welcome to help.

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