Nonlinear electron-hole kinematics in moon wakes

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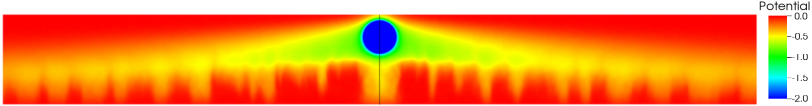
Cross-Field Plasma Wakes

Background plasma drift perpendicular to a strong magnetic field.

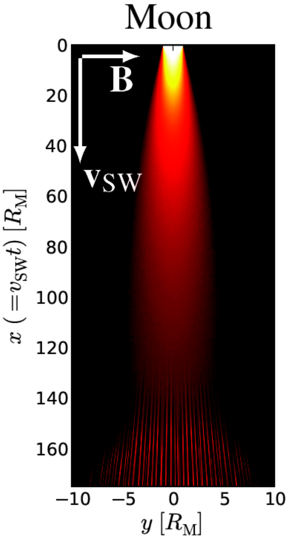


B ⟶

Mach Probe in tokamak edge. Subsonic v_d. $\mathbf{v}_d \downarrow$

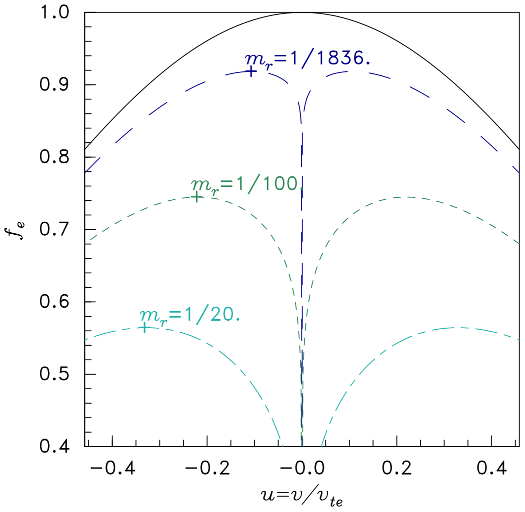
Ions and electrons fill the wake along B: $v_{\|} \longrightarrow$ $\longleftarrow v_{\|}$

Solar-wind wake of the moon. Supersonic $v_d = v_{SW}$.
The focus of this talk.



A dimple forms in $f_e(v)$ by combination of v_d, $v_\parallel$

Dimple electron velocity distributions at the wake potential ridge. They are Langmuir unstable



Distribution calculated by integration along orbits*.

Dimple is greatly enhanced by artificially large electron mass.

Prior simulations with

$$1/m_r \equiv m_i/m_e \ll 1836$$

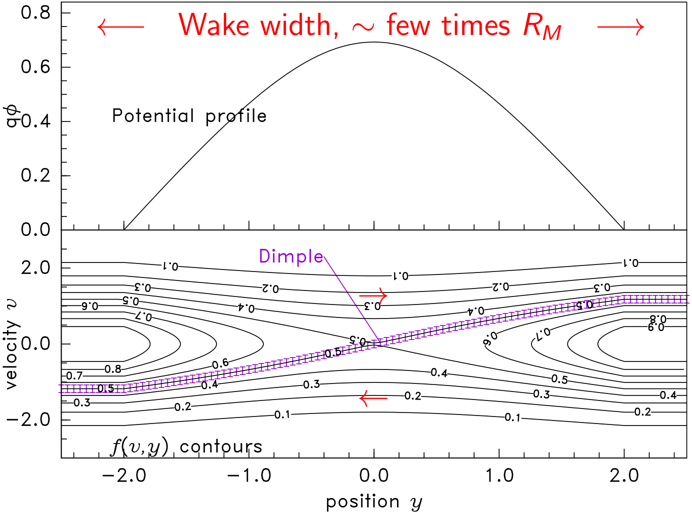
are therefore unreliable.

Distribution is unstable even for physical mass ratio.

* Electron velocity distribution instability in magnetized plasma wakes and artificial electron mass
I. H. Hutchinson, JGR 117, A03101 (2012)

Parallel phase-space of the wake-potential structure

Dimple arises from virtual stagnation near the orbit x-point



Electron potential *energy* profile (schematic).

Total energy is constant on phase space orbits.

The dimple forms as the orbits pass close to the x-point at the potential peak.

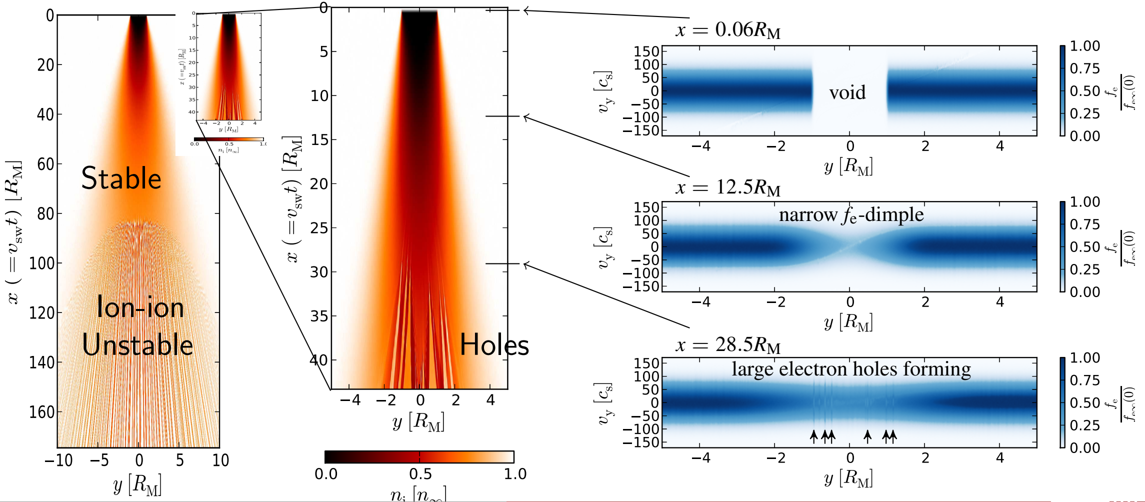
Kinetic-electron simulations disrupt earlier than Boltzmann fluid

1-D PIC simulations in which $v_d t = x$ show instability. Electron dimple present downstream.

Fluid electrons:

Kinetic electrons:

Electron Phase Space:



Electron hole is a self-trapping potential structure

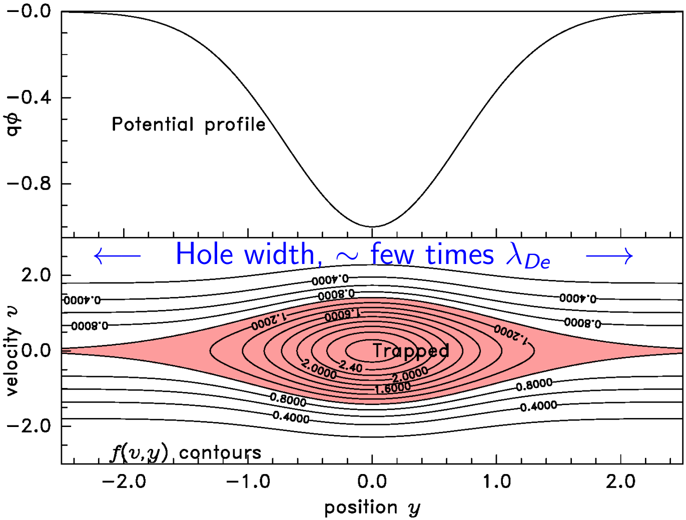
Deficit of electrons causes attractive, electron-trapping, potential energy well

Phase-space-density f_e is a function only of kinetic+potential energy.

On passing orbits, $f_e(v)$ is equal to distant value at same energy.

On trapped orbits, f_e is a "free choice". (BGK mode.)

Typical hole spatial width ∼ few λ_{De}.

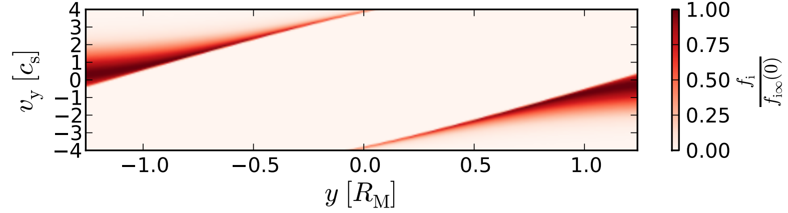


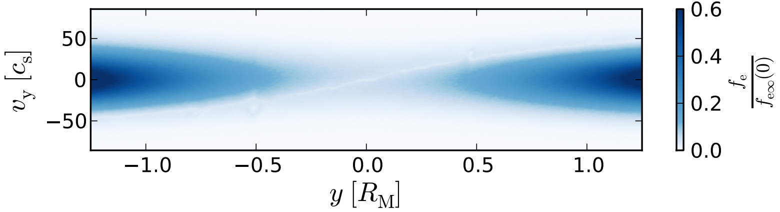
Electron instabilities appear close to separatrix

$\frac{m_{\mathrm{i}}}{m_{\mathrm{e}}} = 459$

$\lambda_{\mathrm{D\,e}} = 0.00125 R_{\mathrm{M}}$

Ion phase-space



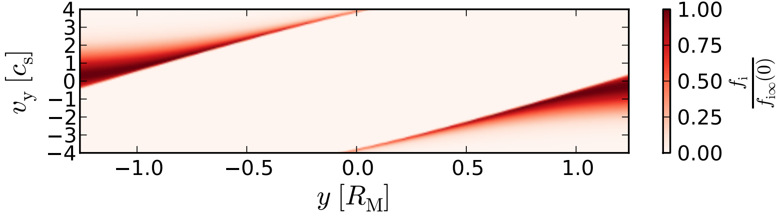


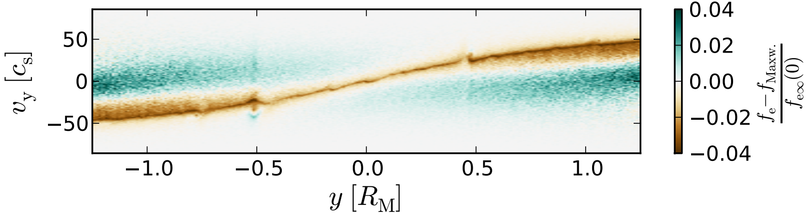
Electron phase-space

Improved visualization by Maxwellian subtraction

$\frac{m_i}{m_e} = 459$ $\lambda_{De} = 0.00125 R_M$

Ion phase-space



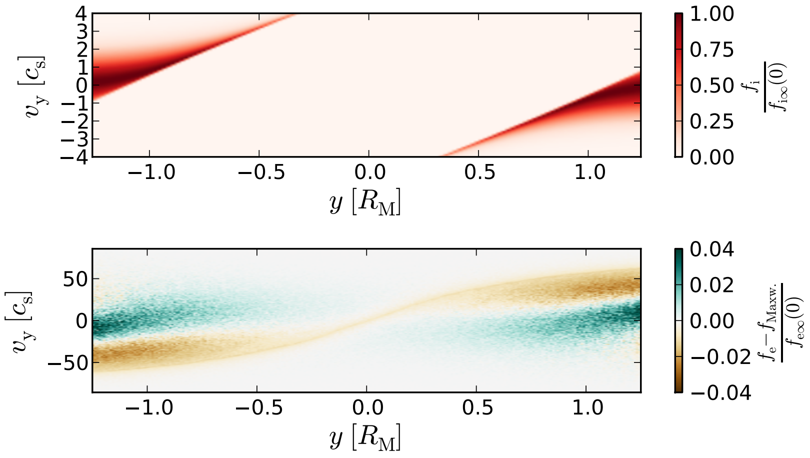


Maxwellian-subtracted electron distribution

Time sequence shows evolution

$\frac{m_\mathrm{i}}{m_\mathrm{e}} = 459$ $\lambda_{\mathrm{D\,e}} = 0.00125 R_\mathrm{M}$

Ion phase-space

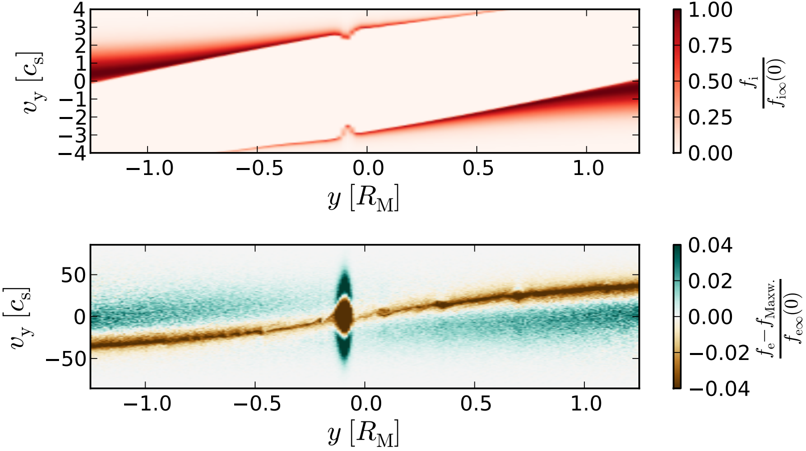


Maxwellian-subtracted electron distribution

Stationary hole grows to large width

$\frac{m_\mathrm{i}}{m_\mathrm{e}} = 459$ $\lambda_{\mathrm{D\,e}} = 0.00125 R_\mathrm{M}$

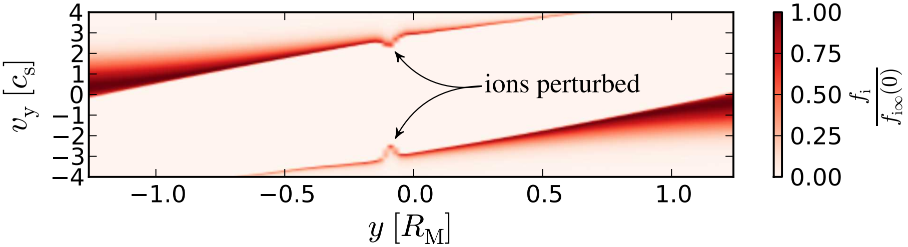
Ion phase-space



Maxwellian-subtracted electron distribution

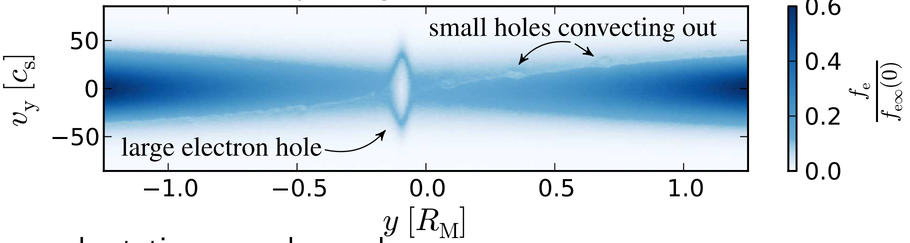
Large electron holes disrupt the ion streams

Ion phase-space distribution



Many small holes convect out before they can grow much.

Electron phase-space



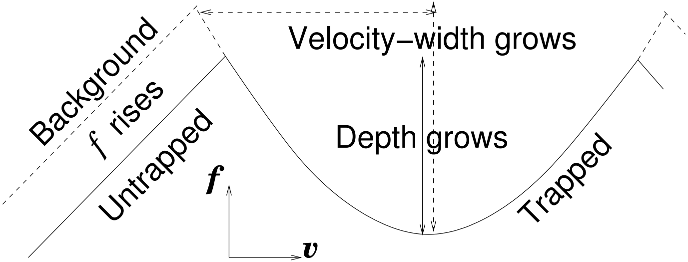
Some holes remain nearly stationary and grow large.

Hole growth mechanism: rise of background f_b

Trapped f-level is fixed. Hole drifts into higher density region.

As f-level surrounding a hole grows, it gets deeper.

A deeper electron hole must also be wider, because $\tilde{f} \propto v_s$.



Therefore drift into a region of higher density causes a hole to grow.

In moon wake, drift to higher f caused the dimple in the first place.

The dimple-forming mechanism is also a hole-growth mechanism.

I H Hutchinson, C B Haakonsen, C Zhou "Non-linear plasma wake growth of electron holes" Physics of Plasmas **22** (2015) 032312

Why are some holes stationary, others move with electrons?

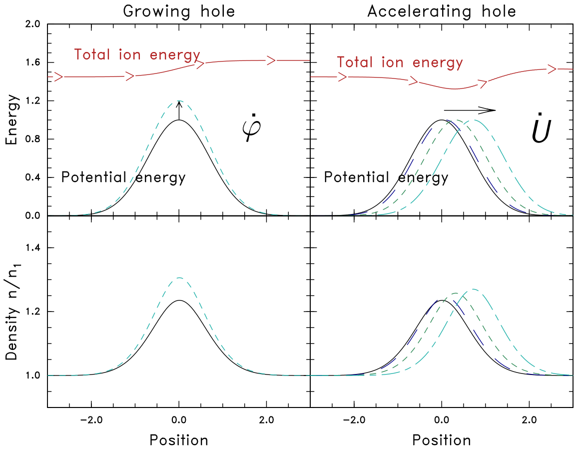
Answer is interaction of ion streams with hole. Requires understanding Hole Kinematics.

Treat the hole as a composite entity with net momentum (and energy).

Hole growth (potential change) $\dot{\varphi}$ or acceleration $\dot{U}$, changes electron and ion momentum/density. Particles are energized. We call this effect **jetting**.

Momentum outflow (passing particles), plus contained momentum, must be conserved.

These determine the Hole Kinematics.



Result **No Overtaking** : Electron holes cannot have same velocity as ion streams.

Complete expressions for momentum change in inertial frame

Outflow, Contained, Background ion momentum rate of change = integral over entering velocity v_1 in hole frame $v=\sqrt{v_1^2+2\varphi/m}$ is velocity at hole position x

$$\dot{P}_{oi}+\dot{P}_{ci}+\dot{P}_{bi}=m_i(\dot{U}-\dot{v}_{bi})\int_{x_1}^{x_2}\int\left[-2+3\frac{v_1}{v}-\left(\frac{v_1}{v}\right)^3\right]f_{1i}(v_1)dv_1\;dx,$$

Expression for electrons requires accounting also for trapped particles

$$\dot{P}_{oe}+\dot{P}_{ce}+\dot{P}_{be}+\dot{P}_{t}=m_e(\dot{U}-\dot{v}_{be})\int_{x_1}^{x_2}\int\left[-1+2\frac{v_1}{v}-\left(\frac{v_1}{v}\right)^3\right]f_{1e}(v_1)dv_1\;dx,$$

Hole growth is outflow plus contained change of momentum, electron & ion

$$\dot{P}_g=m\int_{x_1}^{x_2}\int\left[\left(\frac{v_1}{v}\right)-\left(\frac{v_1}{v}\right)^3\right]\frac{\dot{\varphi}}{mv_1^2}\,f_1(v_1)v_1dv_1dx.$$

Momentum conservation is $\boxed{\sum_{species}(\dot{P}_o+\dot{P}_c+\dot{P}_b+\dot{P}_g)+\dot{P}_t=0}$

Approximations and Simplifications

All expressions above use the "short transit time" approximation,
i.e. acceleration and growth timescales are long compared with particle transit time.

Ion distribution is usually well approximated by a single beam $f_i = n_i\delta(v - v_i)$,
whose kinetic energy $\frac{1}{2}mv^2$ substantially exceeds potential $|\varphi_i|$, then to lowest order[1]

$$\boxed{\dot{P}_o + \dot{P}_c + \dot{P}_c \simeq -3m_i n_1(\dot{U} - \dot{v}_b)\int_{x_1}^{x_2}\left(\frac{\varphi(x)}{m_i v_1^2}\right)^2 dx}, \quad \boxed{\dot{P}_g \simeq -m_i n_1 v_1 \int_{x_1}^{x_2}\frac{\partial}{\partial t}\left(\frac{\varphi(x)}{m_i v_1^2}\right)^2 dx}$$

For Maxwellian **electrons**, and holes slow c.f. v_{te},

$$\boxed{\dot{P}_{eo} + \dot{P}_{ec} + \dot{P}_{eb} + \dot{P}_{et} = -m_e(\dot{U} - \dot{v}_b)n_1\int_{x_1}^{x_2} h(\sqrt{|\varphi|/T_e})dx},$$

where the function h is $h(\chi) = -\frac{2}{\sqrt{\pi}}\chi + \left[(2\chi^2 - 1)\mathrm{e}^{\chi^2}\mathrm{erfc}(\chi) + 1\right], \quad \xrightarrow[\chi\to 0]{} \chi^2, \ \xrightarrow[\chi\to\infty]{} 1$

---

[1] Counterintuitive negative momentum because of contained contained particle density changes.

Kinematics predicts observations of PIC code simulation

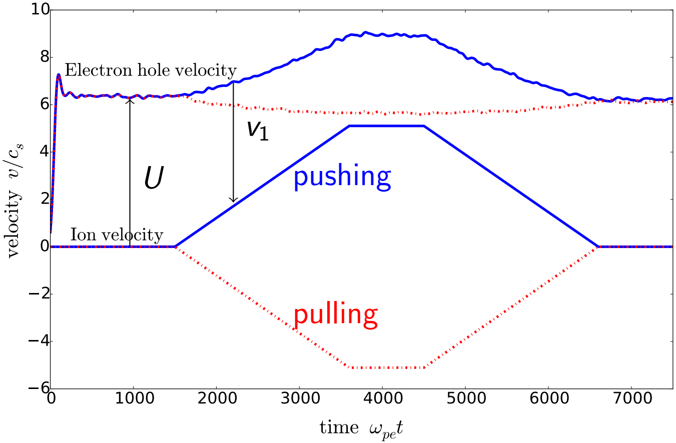
Artificially accelerate ions toward hole velocity (pushing) or away (pulling): by background $\dot{v}_{bi}$.

Hole responds by accelerating: $\dot{U}$.

Agrees quantitatively with integration of Kinematic differential equation:

$$\left[\frac{U}{c_s}\right]_A^B = \left[\frac{M_{ie}^4 c_s^3}{-3v_1^3}\right]_A^B$$

$M_{ie} \sim (\frac{m_i}{m_e})^{1/4}$ is hole mach number at which electron jetting = ion. (Hole initially self-accelerates to it)



Hole velocity is “repelled” by ion stream; but effect is reversible.

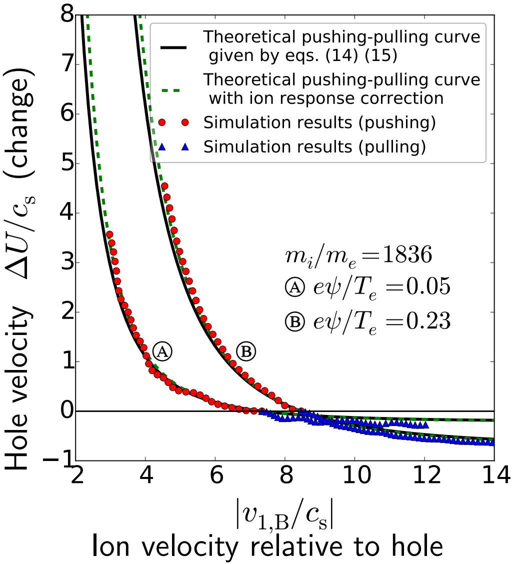
Excellent quantitative agreement with PIC results

Comparison of the Kinematic prediction of hole pushing/pulling with the velocity changes observed in PIC code experiments.

Extremely good agreement is obtained.

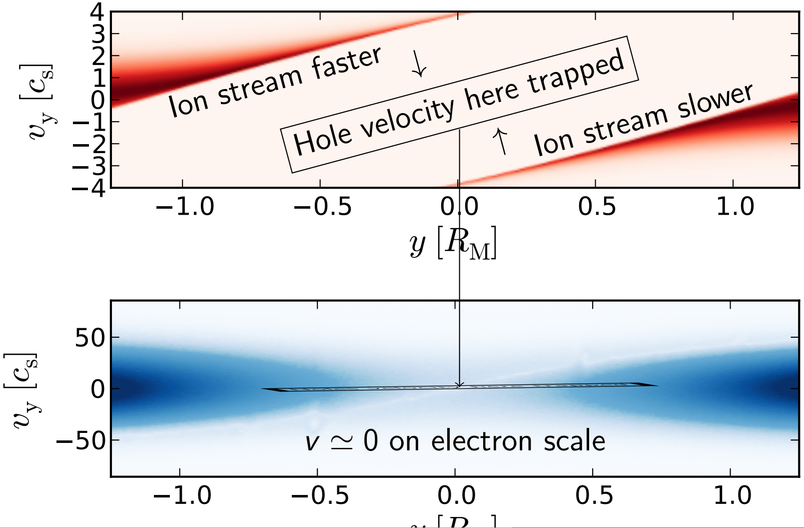
Holes can be pushed to large Mach number.

Ions' velocity does not reach hole's  (v_1 does not change sign.)



Slow holes' velocity trapped between converging ion streams

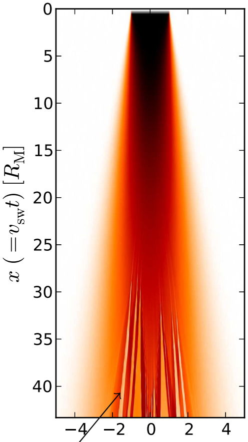
Explains nearly zero speed (c.f. v_{te}), but sloping, wake hole trajectories


Ion stream faster ↓

Hole velocity here trapped

↑ Ion stream slower

$v \simeq 0$ on electron scale


Explains these

Summary

Electron holes in (e.g. moon) magnetized cross-field plasma wake are generated by a newly identified mechanism: Drift into increasing density plasma.

Some holes are trapped with velocity between the two ion streams.

Then they remain and grow very large, disrupting the ion streams.

The trapping mechanism, and much else, is explained by a new theory:
the Kinematics of holes,
verified by PIC simulations of isolated holes.

Supplementary Slides

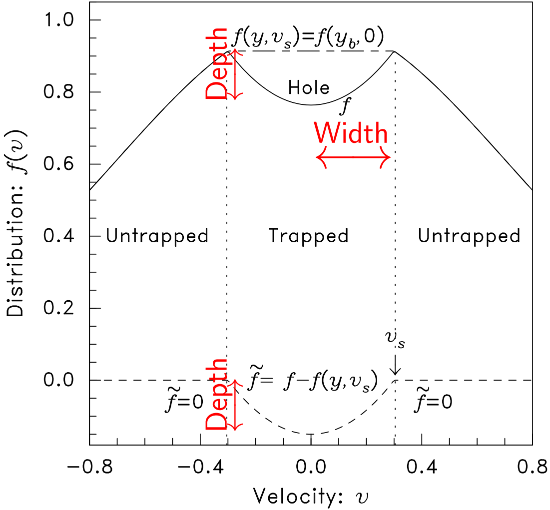
Hole Depth and Width are proportional

In order to satisfy quasineutrality condition

Quasineutrality requires potential-perturbation of untrapped-f to cancel the trapped-f deficit. Result:

$$\overbrace{-\tilde{f}(0)/f_b(0)}^{relative\ \ depth} = \underbrace{\sqrt{\pi}\left(\frac{\lambda_{De}}{\lambda_s}\right)^2}_{\sim 1}\underbrace{v_s/v_{te}}_{width}.$$

[f_b background. λ_s shielding.]



How does ion disruption happen so early?

The puzzle is how electron instability can give potential perturbation large enough

The ion distribution is not 2-stream (ion-ion) unstable until the ion streams are closer in velocity than $\sim \pm c_s$.

Electron distributions have the unstable dimple.

But quasilinear diffusion by incoherent modes would produce marginal stability of electrons before significantly diffusing the ions.

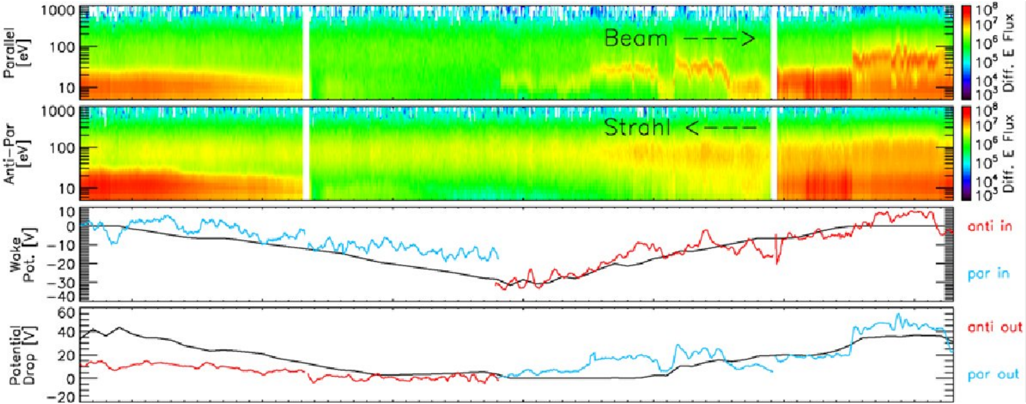
How, therefore, can potential perturbations excited by the dimple grow large enough to disrupt the ions?

Answer: non-linear growth of coherent electron-holes.

Exciting new measurements: Artemis

Spacecraft through the moon's wake, with good plasma diagnostics.

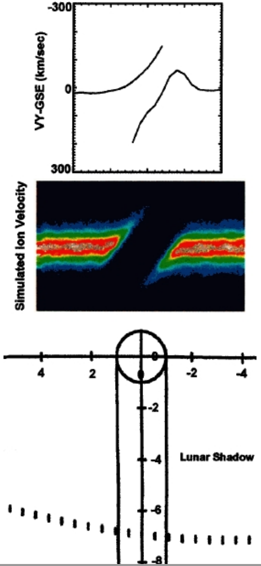
Remarkably detailed electron distribution measurements.

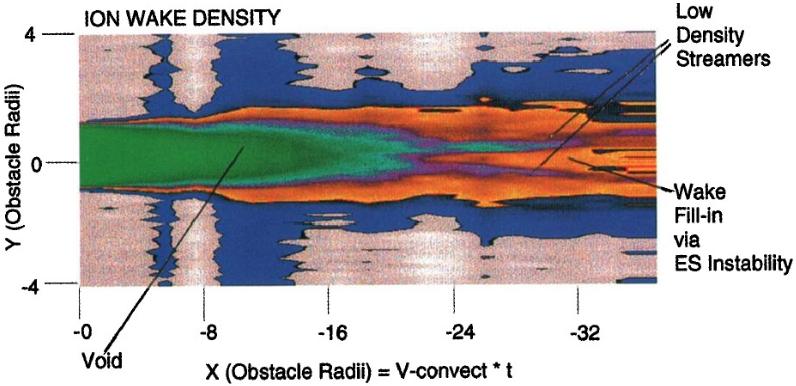


Halekas et al, Space Sci Reviews (2011) DOI10.1007/s11214-010-9738-8

What governs instabilities in a simple, but self-consistent wake?

Example of (1-D, y-t) PIC Simulations



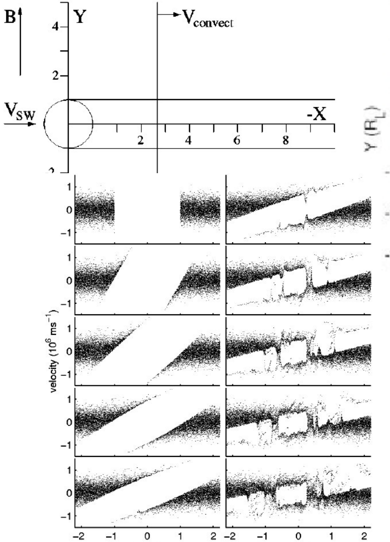


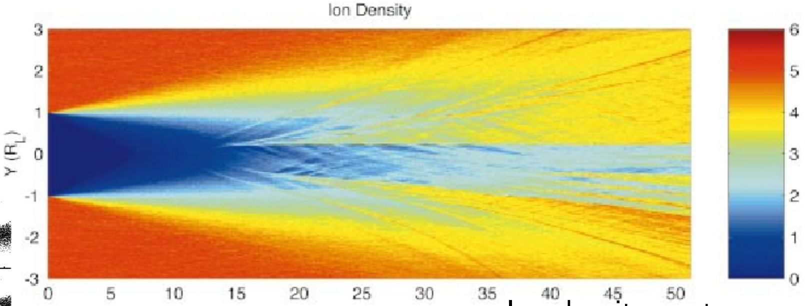
W M Farrell et al, JGR 103, 23653 (1998)

Parameters: $M = 25$, $R_m/\lambda_{De} = 63$ (c.f. 2×10^5 for moon)

$m_i/m_e = 20$ Artificial mass ratio affects instability strongly.

PIC Simulations of Birch and Chapman





Ion density contours

Birch and Chapman, Phys.Plasmas, 8, 4551 (2001)

$M = 25,\ R_m/\lambda_{De} = 128,\ m_i/m_e = 20.$

$\leftarrow$ Ion phase space plots $f(y, v_y)$

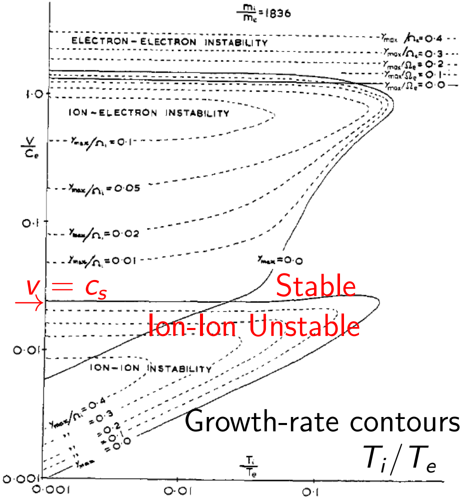
at distances $X/R_m = 0, 3, 6, 9, ..., 27$

Scaled $x = 0.,\ .12,\ .24,\ .36, ...,\ 1.08.$

Classic Gaussian-Beam Analysis (T Stringer 1964)

Ion-ion instability requires beam velocities $< \pm c_s$

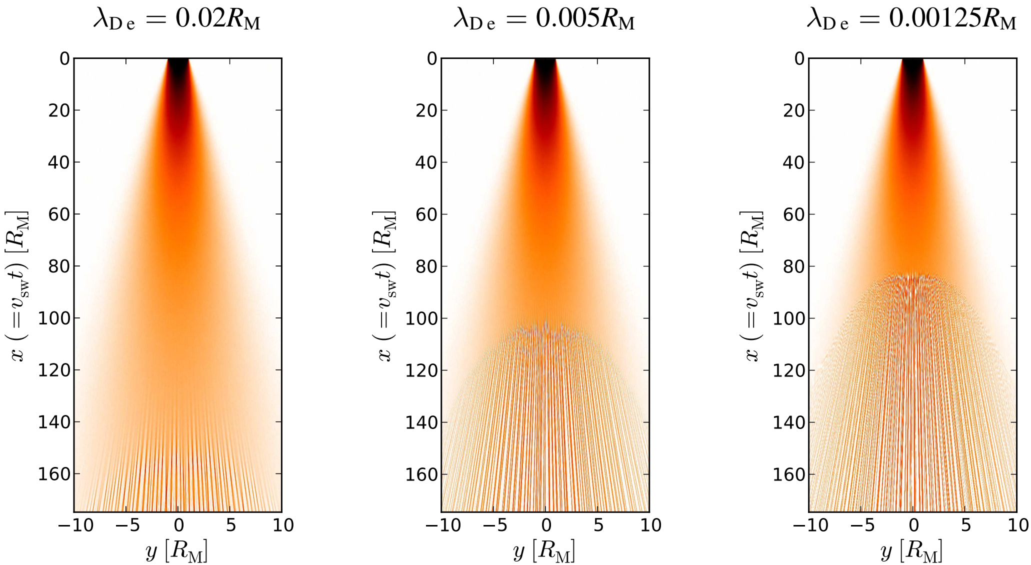
Beam
Velocity



Ion-electron instability is relatively weak and depends upon the electron velocity distribution slope.

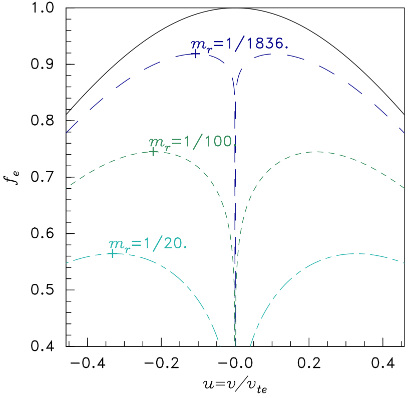
Ion-ion instability (ion 2-stream instability) is strong. But occurs only for (equal and opposite) beam velocities less than c_s.

Instability appears earlier for shorter λ_{De}



Analytic Solution of Dimple on Potential Ridge

$$f_e(v) = \underbrace{\left[\frac{\sqrt{m_r}|v|w}{2\sqrt{|\phi_0|}Y}\right]^P}_{dimple} \times \underbrace{\frac{n_\infty}{\sqrt{2\pi T}}\exp(\phi_0)\exp(-\frac{1}{2}m_r v^2)}_{Boltzmann}$$



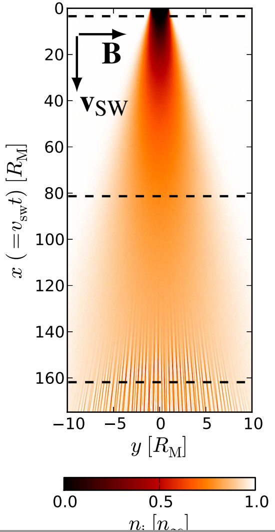
where v and ϕ_0 are the velocity and potential at the ridge ($y = 0$) in normalized units, w and Y are scale-length parameters of order unity, $m_r = m_e/m_i$, and $P = \frac{d\phi_0}{kdx} = \frac{d\phi_0}{dx}\frac{w}{\sqrt{|\phi_0|}}\sqrt{m_r} \sim \sqrt{m_r}$.

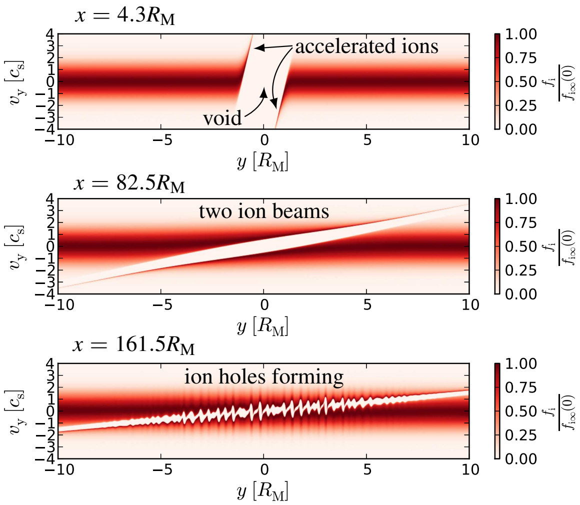
The leading factor is the dimple modification $\sim [\sqrt{m_r}v]^{\sqrt{m_r}}$ giving **dimple velocity half-width**

$$v_{width} \sim c_s/m_r^{1/4} \approx 7c_s.$$

PIC simulation with Boltzmann (Fluid) Electrons

1-D phase-space (y), velocity (v_y). Constant downward $v_x \Rightarrow t \equiv x$-wake-distance.



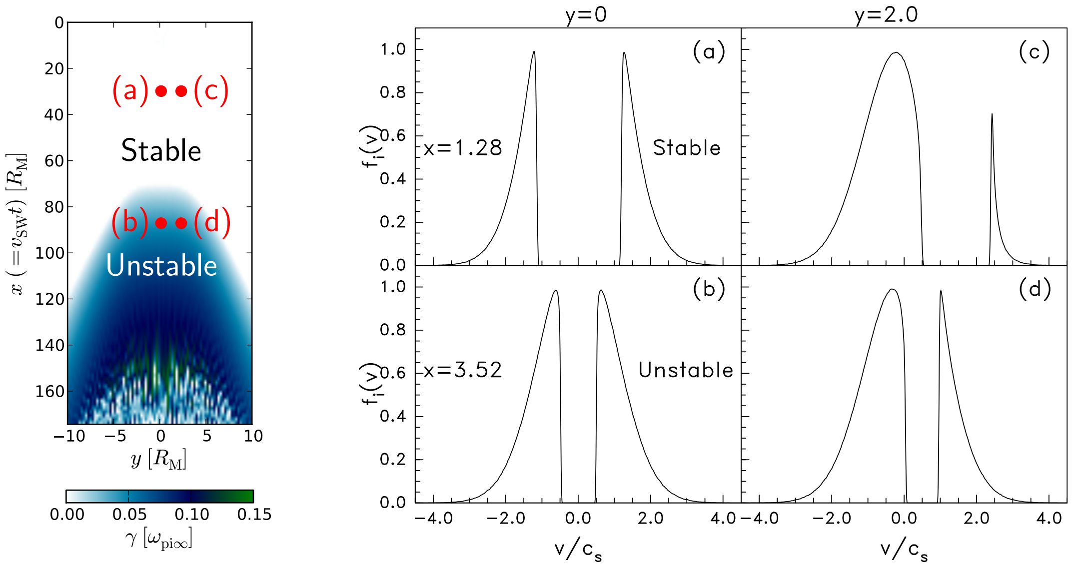


$x = 4.3R_{\rm M}$

$x = 82.5R_{\rm M}$

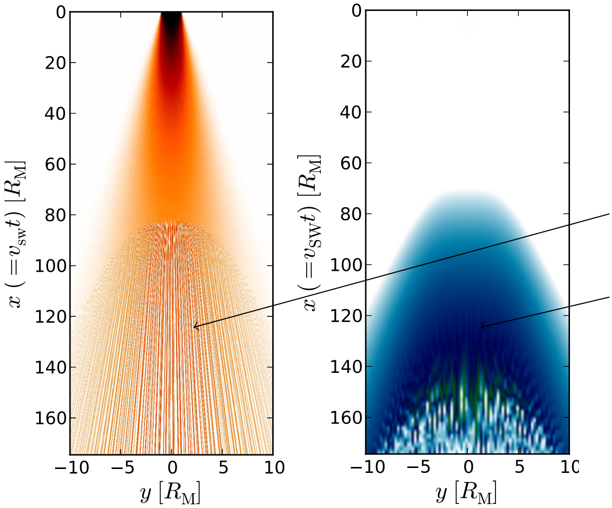
$x = 161.5R_{\rm M}$

Ion distributions from Hybrid PIC Simulation (Fluid electrons)



Hybrid Simulation sees wake ion-ion (not electron) instability.

$\lambda_{\mathrm{D\,e}} = 0.00125 R_{\mathrm{M}}$



Results consistent with linear stability.

Appearance of instability in simulation (left)

Explained by ion-ion linear growth rate (right)

Electron kinetic effects excluded.

$M_{sw} = 25$ sets x-scale.

Summary. A new electron hole growth mechanism is the cause of cross-field wake ion disruption.

A comprehensive explanation of the (numerically) observed 1-d stability behavior of the moon wake has been developed.

Two ion streams are formed but are linearly stable till far downstream.

Electron dynamics produces an unstable dimple in f_e early.
It spawns coherent electron holes (not just quasilinear diffusion).

The holes grow, driven by same mechanism that causes the dimple:
cross-field drift into increasing background density regions.
Holes that remain near the potential ridge grow large enough to tap the ion free energy and explode, disrupting the ion streams.

Mach probes generate instabilities. Does this affect their results?