Computational Physics

21-09-2018

The equations of motion of a point thrown in a viscous fluid from the origin of a Cartesian coordinates system are:

$$\begin{aligned} \frac{d^2x}{dt^2} &= -k\frac{dx}{dt} \\ \frac{d^2y}{dt^2} &= -k\frac{dy}{dt} - g \end{aligned}$$

where x and y represent the position of the point in the Cartesian frame of reference during the motion, k is the friction coefficient of the fluid and $g = 9.81$ is the Earth's gravitational acceleration (all quantities are in MKSA units).

Appropriate initial conditions for the equations are:

$$\begin{aligned} x(t=0) &= 0 \\ y(t=0) &= 0 \\ \left.\frac{dx}{dt}\right|_{t=0} &= v_0 \cos\theta \\ \left.\frac{dy}{dt}\right|_{t=0} &= v_0 \sin\theta \end{aligned}$$

where v_0 is the modulus of the initial velocity of the point and θ is the angle under which the point is thrown.

Solve numerically with a second order Runge-Kutta time scheme the system of equations above up to the time in which the point reachs the ground ($y = 0$) for at least five values of k in the range: $k \in [0, 1]$ and fixed values of v_0 and θ and study how the range (distance from the origin to the point where the body falls on the ground) depends on the value of k. Afterwards, chosen a single value for k and θ, study how the range depends on v_0 (for at least 5 values of v_0 in the range $v_0 \in [1, 5]$). Finally, choose a value for k and v_0 and study how the range depends on θ by varying continously its value in the range $\theta \in [0°, 90°]$ and determine approximately for which value of θ one gets the maximum range.

For comparison, one can check the correctness of the results for x and y with the analytical results:

$$\begin{aligned} x(t) &= \frac{v_0 \cos\theta}{k}(1 - e^{-kt}) \\ y(t) &= \frac{v_0 \sin\theta}{k}(1 - e^{-kt}) - \frac{g}{k}\left[\frac{1}{k}\left(1 - e^{-kt}\right) - t\right] \end{aligned}$$