Computational Physics

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Lorenz's model is a simplified model for atmospheric convection, under the form of a system of total differential equations:

$$\begin{aligned}
\frac{dx}{dt} &= \sigma(y-x) \\
\frac{dy}{dt} &= \rho x - xz - y \\
\frac{dz}{dt} &= xy - \beta z
\end{aligned}$$

where x, y, z represent (in dimensionless units) the velocity of convective motions in the vertical plane, the difference of temperature between the ascending and descending masses and the fluctuations of temperature with respect to the background, respectively. The parameters σ, ρ and β are connected to average quantities of the system.

Lorenz realized (just by chance!) that the system of equations exhibits a strong sensitivity to initial conditions, namely that solutions of the system obtained for initial conditions very close each to the other can show a completely different behaviour, that is the system is *chaotic*.

Solve numerically the Lorenz's system of equations above with a second order Runge-Kutta time scheme, for the following values of the parameters:

$$\begin{aligned}
\sigma &= 10 \\
\rho &= 28 \\
\beta &= \frac{8}{3}
\end{aligned}$$

and with initial conditions given by: $x(0) = 1$, $y(0) = 11$, $z(0) = 20$ and $x(0) = 0.999$, $y(0) = 11$, $z(0) = 20$, for a time interval $t = [0, 20]$, and by using a time step $h = 10^{-5}$. The students should point out as, for $t \gtrsim 8$, the trajectories of the solutions separate each to the other, by assuming a chaotic behaviour. Finally, they should study the shape of the trajectories in the phase space: $x(t)$-$y(t)$, $x(t)$-$z(t)$ and $y(t)$-$z(t)$.